



**INSA** INSTITUT NATIONAL  
DES SCIENCES  
APPLIQUÉES  
LYON



## 18th European Mechanics of Materials Conference (EMMC18)

# A NEW APPROACH TO ACCOUNT FOR THE STEEL-CONCRETE INTERFACE IN CASE OF INDUSTRIAL STRUCTURE APPLICATIONS USING KINEMATIC PROJECTION

Maryam Trad<sup>1</sup>, Ibrahim Bitar<sup>1</sup>, Stéphane Grange<sup>2</sup>, and Benjamin Richard<sup>1</sup>

<sup>1</sup> Institut de Radioprotection et de Sécurité Nucléaire (IRSN), PSN-EXP/SES/LMAPS, B.P. 17 - 92262 Fontenay-aux-Roses Cedex, France

<sup>2</sup> Univ Lyon, INSA Lyon, GEOMAS, EA7495, 69621 Villeurbanne, France

[maryam.trad@irsn.fr](mailto:maryam.trad@irsn.fr)

[ibrahim.bitar@irsn.fr](mailto:ibrahim.bitar@irsn.fr)

[stephane.grange@insa-lyon.fr](mailto:stephane.grange@insa-lyon.fr)

[benjamin.richard@irsn.fr](mailto:benjamin.richard@irsn.fr)

# Presentation outline



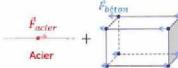
Interest of the PhD subject: Energy dissipation at the steel-concrete interface level.



Physical description of the interface behaviour.



Experimental characterization of the steel-concrete bond.



Numerical strategies proposed in the literature for the interface modeling.

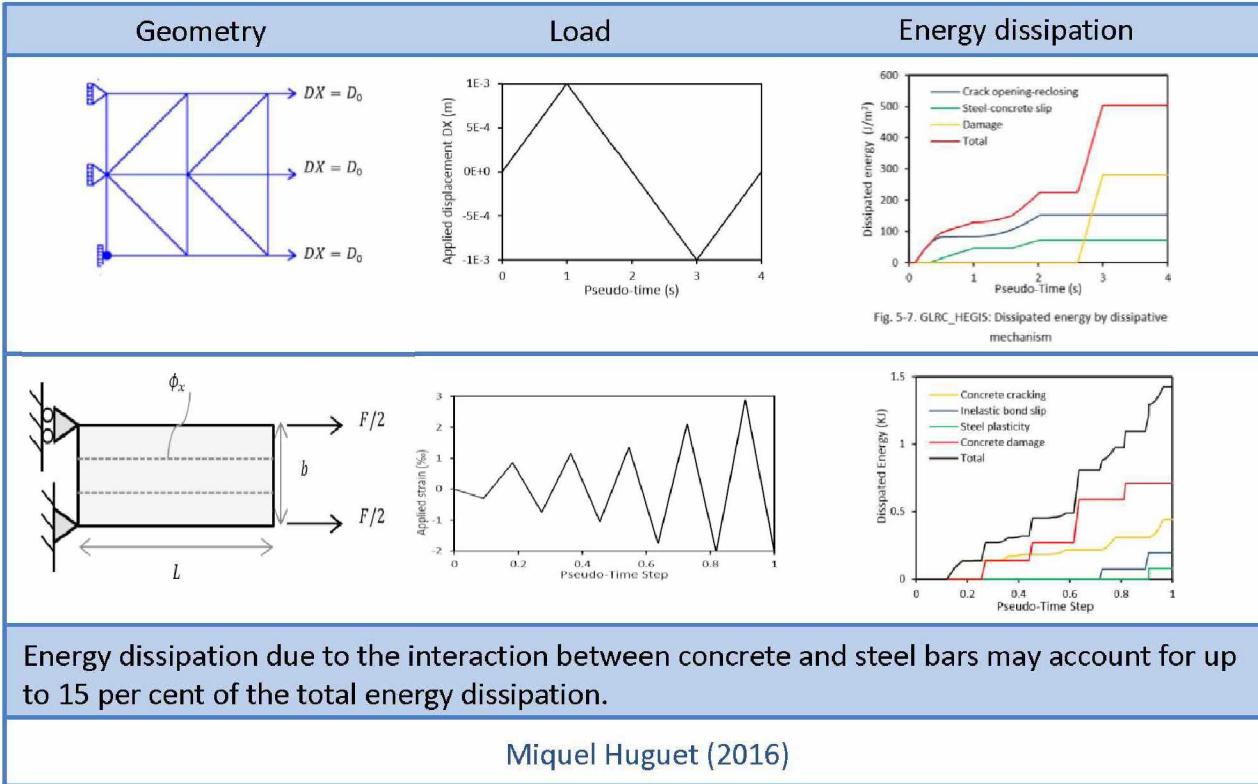


Suggestion of a new modelling strategy to take into account the behaviour of the steel-concrete interface.



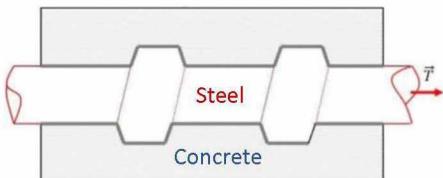
Implementation of the methodology in a beam element.

# Energy dissipation at the steel-concrete interface level

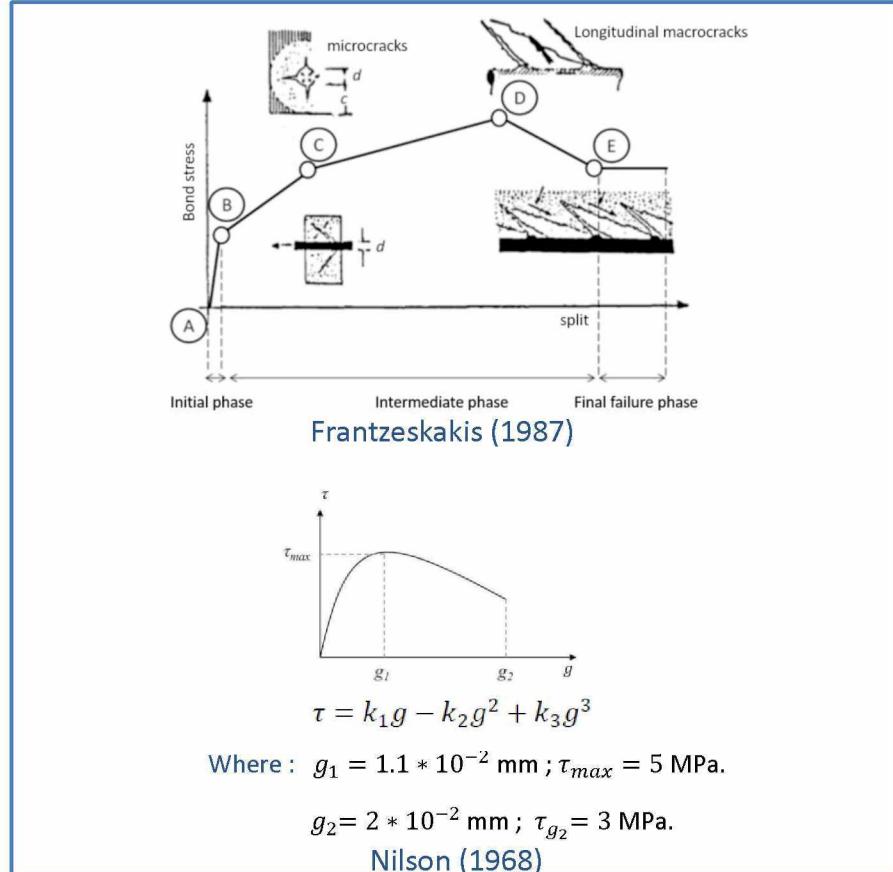


# Physical description of the interface behaviour

- Stage AB:** Bond efficiency is ensured by chemical adhesion between the two materials.
- Stage BC:** This stage is characterized by the occurrence of microcracks surrounding the steel bar.
- Stage CD:** The slip value between steel and concrete increases and longitudinal macrocracks start to appear at D.
- Stage DE:** The bond stress starts to decrease where the slip value continues to increase.
- Stage E:** This is the final failure phase.

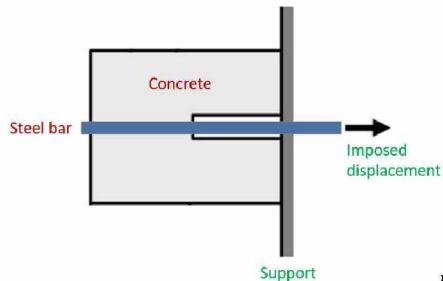


Torre-Casanova (2012)



# Experimental tests

## Pull-out



$$\text{Dahou et al.(2009)} \quad \tau_s = \frac{F}{\varphi \pi l}$$

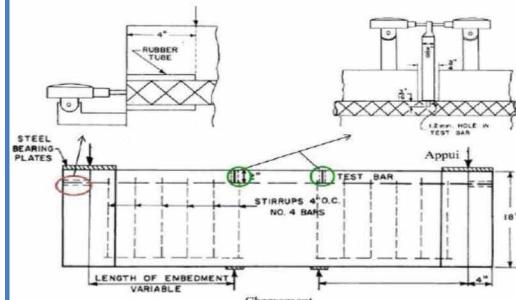
## Ties



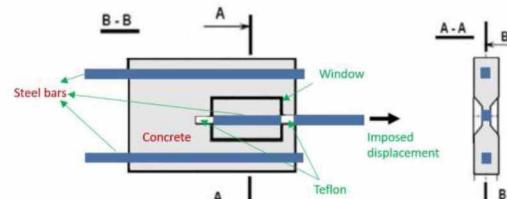
Clément (1987)

Torre-Casanova (2012)

## Beams

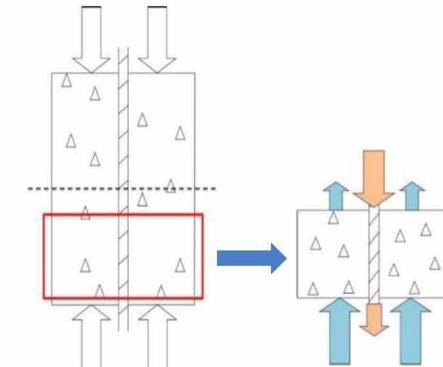
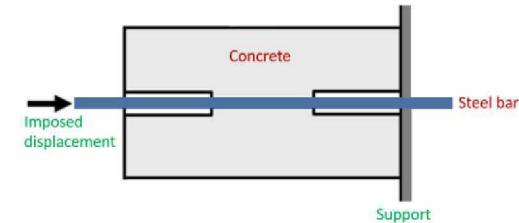


## PIAF test



Tran et al. (2009)

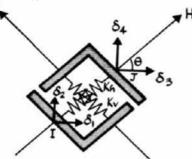
## Push-in



Tixier(2015)

# Numerical modeling approaches

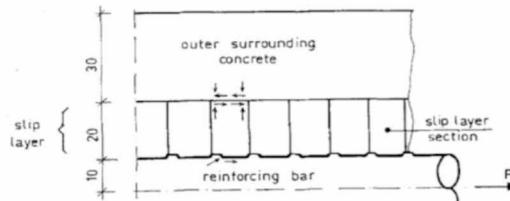
## Spring (bar) elements



$$\begin{pmatrix} \sigma_h \\ \sigma_v \end{pmatrix} = \begin{pmatrix} k_h & 0 \\ 0 & k_v \end{pmatrix} \begin{pmatrix} \epsilon_h \\ \epsilon_v \end{pmatrix}$$

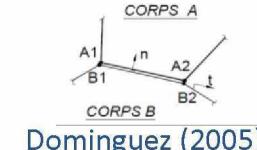
Ngo et Scordelis (1967)

## 2D and 3D explicit area



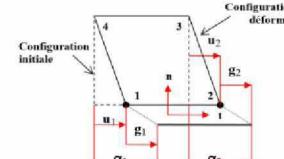
Reinhardt et al. (1984)

### Joint element a with zero thickness value



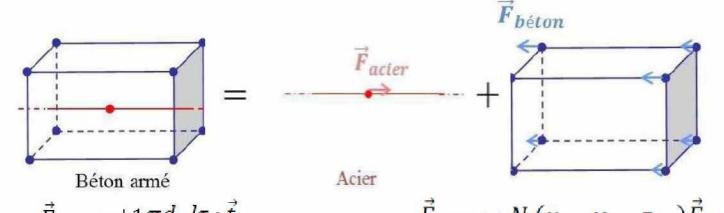
Dominguez (2005)

## Extended finite elements method XFEM



Dominguez (2005)

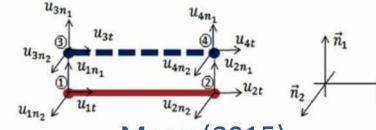
## Introduction of internal forces



$$\vec{F}_{b/a} = \pm 1 \text{ N}$$

Torre-Casanova (2012)

Finite elements with 4 nodes (QUA4)



Mang (2015)

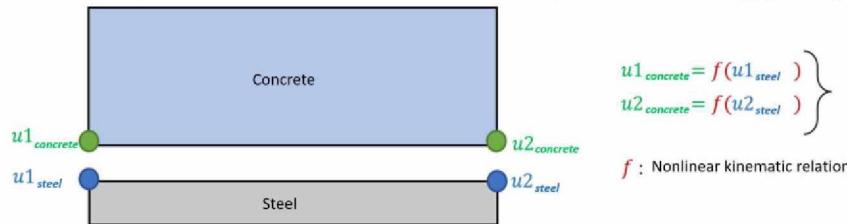
### Limitations

These models are in general applied in detailed 2D and 3D analysis in order to represent some structural elements. Applying these methodologies for large scale structures is highly time consuming.

# Suggestion of a new modelling strategy for steel-concrete interface : concept

## Concept of the proposed strategy

- The main interest is the simplicity of the proposed approach and the possible transition to a large-scale application. This thesis is a direct continuation of the previous works of Torre-Casanova (2012) and Mang (2015).



- The bond-slip law frequently used in the litterature to represent the interface will be replaced by a kinematic nonlinear relation. This relation links the longitudinal displacements of steel and concrete nodes in a finite elements simulation.
- In order to identify the analytical expression of kinematic relations, numerical pull-out tests with kinematic relations representing the interface are to be modeled. Numerical results can be calibrated with respect to experimental ones.

## Advantages

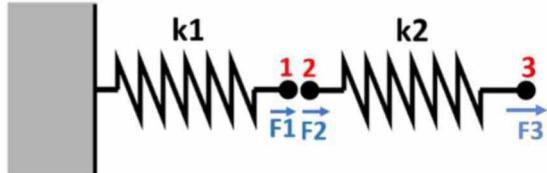
- There is no need to add additional degrees of freedom to the studied problem in order to take into account the numerical modeling of the interface.
- It is easy to integrate the methodology into a conventional finite elements resolution strategy.

## Implementation

- Single and double Lagrange multipliers.
- Kinematic projection method.

# Suggestion of a new modelling strategy for steel-concrete interface : validation

## Validation example



$$\begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$K \quad U = F$

+

$$u_1 = \alpha u_2 \equiv \underbrace{\begin{bmatrix} 1 & -\alpha & 0 \end{bmatrix}}_L \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = 0$$

$L$  : constraints matrix

## Simple Lagrange multipliers

$$KU + L^T \lambda = F$$

$$LU = d$$

$$\boxed{\begin{bmatrix} k & L^T \\ L & 0 \end{bmatrix}} \begin{bmatrix} U \\ \lambda \end{bmatrix} = \begin{bmatrix} F \\ d \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{1}{k_1} & \frac{1}{\alpha k_1} & \frac{1}{\alpha k_1} & 0 \\ \frac{1}{\alpha k_1} & \frac{1}{\alpha^2 k_1} & \frac{1}{\alpha^2 k_1} & -\frac{1}{\alpha} \\ \frac{1}{\alpha k_1} & \frac{1}{\alpha^2 k_1} & \frac{1}{\alpha^2 k_1 + k_2} & -\frac{1}{\alpha} \\ 0 & -\frac{1}{\alpha} & -\frac{1}{\alpha} & 0 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \end{bmatrix}$$

## Double Lagrange multipliers

$$\boxed{\begin{bmatrix} k & L^T & L^T \\ L & -\beta I & \beta I \\ L & \beta I & -\beta I \end{bmatrix}} \begin{bmatrix} U \\ \lambda^1 \\ \lambda^2 \end{bmatrix} = \begin{bmatrix} F \\ d \\ d \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \lambda^1 \\ \lambda^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{k_1} & \frac{1}{\alpha k_1} & \frac{1}{\alpha k_1} & 0 & 0 \\ \frac{1}{\alpha k_1} & \frac{1}{\alpha^2 k_1} & \frac{1}{\alpha^2 k_1} & -\frac{1}{2\alpha} & -\frac{1}{2\alpha} \\ \frac{1}{\alpha k_1} & \frac{1}{\alpha^2 k_1} & \frac{1}{\alpha^2 k_1 + k_2} & -\frac{1}{2\alpha} & -\frac{1}{2\alpha} \\ \frac{1}{\alpha k_1} & \frac{-1}{2\alpha} & \frac{-1}{2\alpha} & \frac{1}{4\beta} & \frac{1}{4\beta} \\ 0 & \frac{-1}{2\alpha} & \frac{-1}{2\alpha} & \frac{1}{4\beta} & \frac{-1}{4\beta} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \\ 0 \end{bmatrix}$$

# Suggestion of a new modelling strategy for steel-concrete interface : validation

## Kinematic projection method

### Concept

Degrees of freedom are divided into two groups : dependent and independent degrees. A projection matrix is introduced. It links the incremental values of dependent and independent degrees. The equilibrium equation is multiplied by the transpose of this projection matrix.

### Implementation

$u_1 = \alpha u_2$  ;  $u_1$  : dependent,  $u_2$  and  $u_3$  : independent degrees.

$$\delta U = P \delta U_{libres} ; \begin{bmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta u_2 \\ \delta u_3 \end{bmatrix} = P \begin{bmatrix} \delta u_2 \\ \delta u_3 \end{bmatrix} ; U^* = U : \text{virtual degrees of freedom vector.}$$

PVW :  $\delta U^{*T} KU = \delta U^{*T} F$  ;  $\delta U_{libres}^{*T} P^T KU = \delta U_{libres}^{*T} P^T F$  ; So :  $P^T KU = P^T F$  (for linear kinematic relations  
 $\delta U = P \delta U_{libres}$  and  $U = PU_{libres}$ ).

### Application

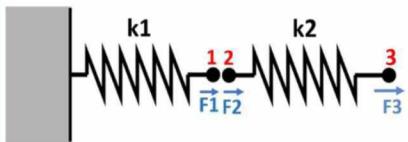
$$P^T k P = \begin{bmatrix} \alpha & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} ; P^T F = \begin{bmatrix} \alpha^2 k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} \alpha F_1 + F_2 \\ F_3 \end{bmatrix} ; \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \alpha^2 k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}^{-1} \begin{bmatrix} \alpha F_1 + F_2 \\ F_3 \end{bmatrix}$$

## Conclusion

The kinematic projection methodology will be adopted because it reduces the size of the problem to be resolved.

# Suggestion of a new modelling strategy for steel-concrete interface : validation

Example

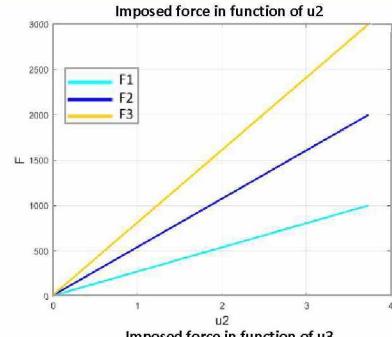


With :

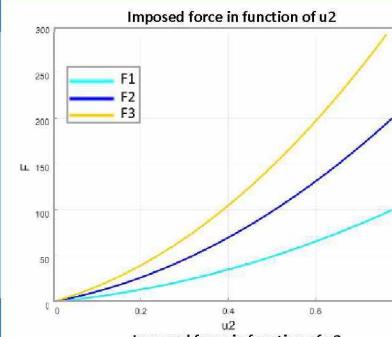
$$k_1 = 40;$$

$$k_2 = 80.$$

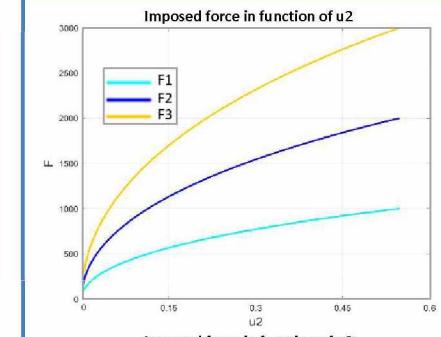
$$u_1 = 0.1 + 10u_2$$



$$u_1 = 3u_2 + 3u_2^2$$



$$u_1 = 40\sqrt{u_2}$$



$$\begin{bmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 10 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_P \begin{bmatrix} \delta u_2 \\ \delta u_3 \end{bmatrix}$$

$$\begin{bmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 3 + 3u_2 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_P \begin{bmatrix} \delta u_2 \\ \delta u_3 \end{bmatrix}$$

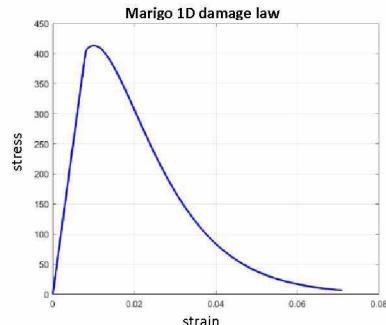
$$\begin{bmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 20 & 0 \\ \sqrt{u_2} & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_P \begin{bmatrix} \delta u_2 \\ \delta u_3 \end{bmatrix}$$

# Suggestion of a new modelling strategy for steel-concrete interface : validation

Marigo 1D damage law

$$f = Y - Y_D - SM d$$

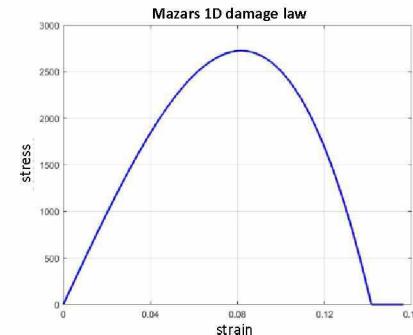
- $Y = \frac{1}{2} E \varepsilon^2$  : elastic energy recovery rate.
- $SM$  : the evolution of  $f$  in function of the damage.
- $Y_D$  : initial damage threshold,
- $d$ :damage internal variable.



Mazars 1D damage law

$$f = \varepsilon_{eq} - \varepsilon_{d0}$$

- $\varepsilon_{d0}$  : strain elastic limit.
- $\varepsilon_{eq} = \sqrt{\langle \varepsilon \rangle_+ : \langle \varepsilon \rangle_+}$  .

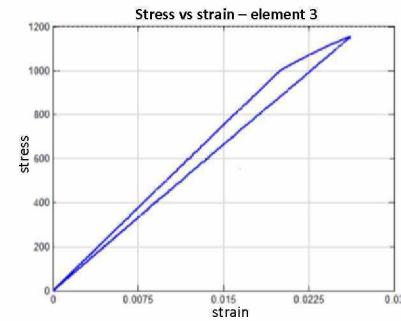
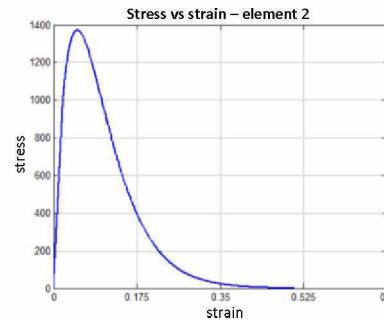
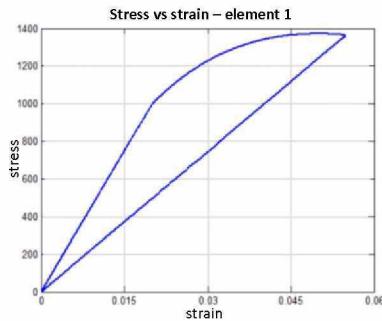
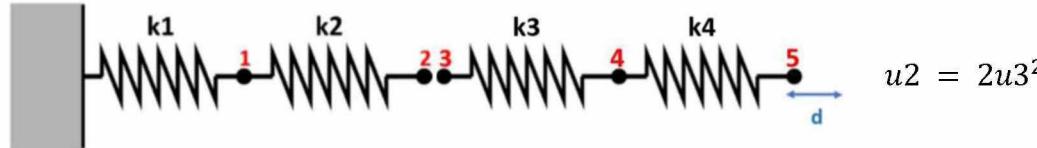


Objective

The aim of the incorporation of damage laws in the 1D validation examples is to test the possibility to couple two types of nonlinearities in one simulation: nonlinear kinematic relations, and nonlinear material behaviour laws.

# Suggestion of a new modelling strategy for steel-concrete interface : validation

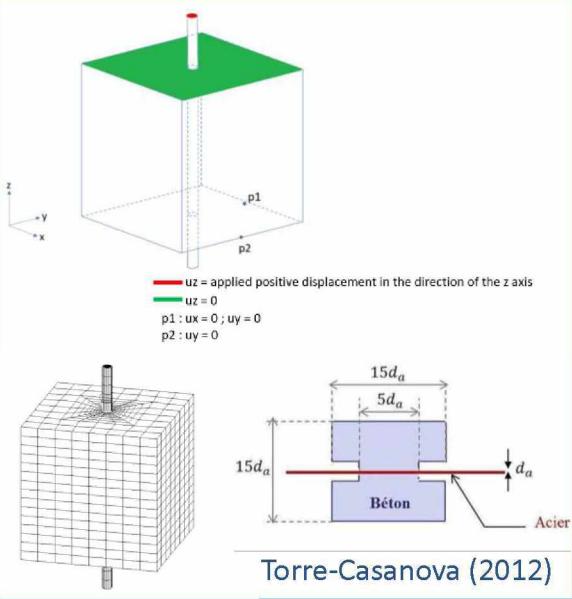
## Validation example with two types of nonlinearities



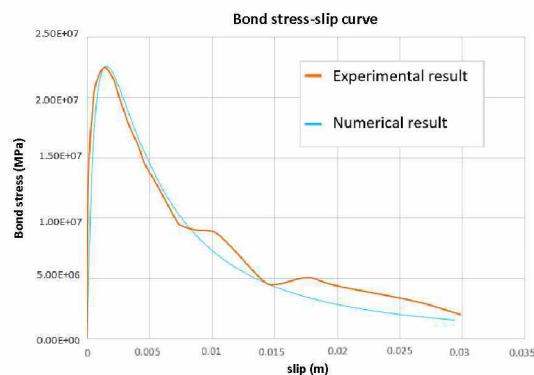
Identical result for element 4.

# Suggestion of a new modelling strategy for steel-concrete interface : application

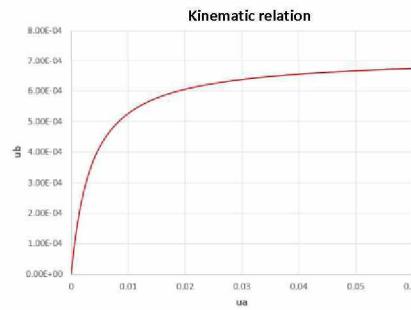
Mesh and boundary conditions



Calibrated result



Kinematic relation



$$u_b = \frac{u_a}{a + bu_a}$$

Where:

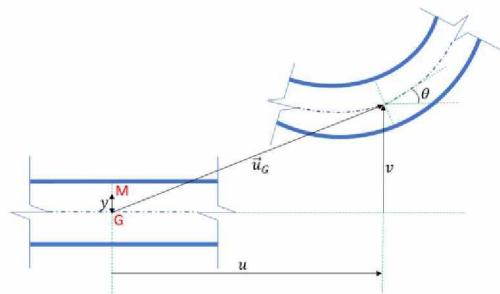
$$a = 5 ; b = 1500.$$

Conclusions

- These results are an evidence that kinematic relations can be used to represent the steel-concrete interface. In fact, using kinematic relations means in other terms that the slip ( $u_a - u_b$ ) between the two materials is being imposed.
- The next step for this pull-out example will be the use of nonlinear damage laws for concrete.

# Implementation in a beam element

## Classic Euler-Bernoulli beam formulation

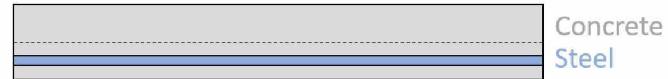


$$\vec{u}_M = \vec{u}_G + \theta \wedge \overrightarrow{GM} = \begin{bmatrix} u - y\theta \\ v \end{bmatrix}$$

$$\varepsilon_M(x) = [1 \quad -y] \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial \theta} \end{bmatrix} = [1 \quad -y] \begin{bmatrix} [B1] \\ [B3] \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix} = [1 \quad -y] B_{13} u_e$$

Where the matrix  $B_{13}$  has the derive values of the function forms.  $B_{13}$  links the strain of an eccentric point M of the beam section to the strain of point G located in the same beam section, on its center line.

## Introduction of slip deformation



$$\varepsilon_a = [1 \quad -y] B_{13} u_e + \Gamma [1 \quad -y] B_{13} u_e^a$$

The idea is to introduce a slip deformation between steel and concrete. The classic beam theory imposes an identical strain between the two materials.

Because of the modification of the kinematic relations of the classical beam formulation, the beam section will no longer be straight. This means that the steel bar won't be controlled by the rigid body movement of the section. For this reason, the bar is forced to slip with respect to concrete.

This study is in progress.

## Conclusions

- The kinematic projection method has the advantage of reducing the size of the system to be solved.
- The pull-out numerical modelling shows that it is possible to represent the steel-concrete interface by a kinematic relationship in order to reproduce the experimental result expressed in terms of bond stress-slip.

## Perspectives

- Introduction of a non-linear concrete behavior law for the pull-out example.
- Study on the effect of the material and geometric parameters of the pull-out test on the kinematic relationship expression.
- Implementation of the methodology within beam and plate elements.
- Implementation of the methodology in dynamic analysis.

**Thank you for your attention !**

## References

- | A.Torre-Casanova. Prise en compte de la liaison acier-béton pour le calcul de structures industrielles, Ecole normale supérieure de Cachan - ENS Cachan, PhD-2012.
- | C.Frantzeskakis. Contribution à la modélisation des structures en béton armé par la méthode des éléments finis, Ecole nationale des ponts et chaussées, PhD- 1987.
- | A.H. Nilson. Nonlinear Analysis of Reinforced Concrete by the Finite Element Method, *ACI Journal*, vol 65, p. 757-766, Septembre 1968.
- | Z.Dahou, Z.M.Sbarta, A. Castel, F. Ghomari. Artificial neural network model for steel-concrete bond prediction, *Engineering structures* vol 31, p. 1724-1733, 2009.
- | J.-L. Clément, "Interface acier-béton et comportement des structures en béton armé : caractérisation, modélisation," *Université Paris 6*, 1987.
- | B.H. Tran, Y. Berthaud, F. Ragueneau. Essai PIAF: pour identifier l'adhérence et le frottement, AUGC XXVIIeme Rencontres Universitaires de Genie Civil, 2009.
- | A.Tixier. Analyse du comportement de l'interface acier-béton par essai push-in. Mesures par fibres optiques et modélisation par éléments finis, Université de Grenoble, PhD-2013.
- | D.Ngo, A. C. Scordelis. Finite Element Analysis of Reinforced concrete beams *ACI Journal*, Mars 1967.
- | H. W. Reinhardt, J. Blaauwendaard, E. Vos. Prediction of bond between steel and concrete by numerical analysis *Matériaux et Constructions*, 2004.
- | N. Dominguez Ramirez. Etude de la liaison acier-béton: De la modélisation du Phénomène à la Formulation d'un Elément Fini Enrichi "Beton-Armé", Laboratoire de mécanique et technologie, Ecole normale supérieure de Cachan - ENS Cachan, PhD-2005.
- | C.Mang. Modélisation de la liaison acier-béton dans le calcul de structures en béton armé, Université Paris Ouest, PhD- 2015.