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INSTITUT DE RADIOPROTECTION ET DE SÛRETÉ NUCLÉAIRE

VARIANCE REDUCTION FOR MONTE CARLO CRITICALITY CALCULATIONS USING ADAPTIVE MULTILEVEL SPLITTING

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MEMBRE DE ETSON

Motivations

Can we tweak the Adaptive Multilevel Splitting (shielding) to compute a flux distribution in multiplicative media ?

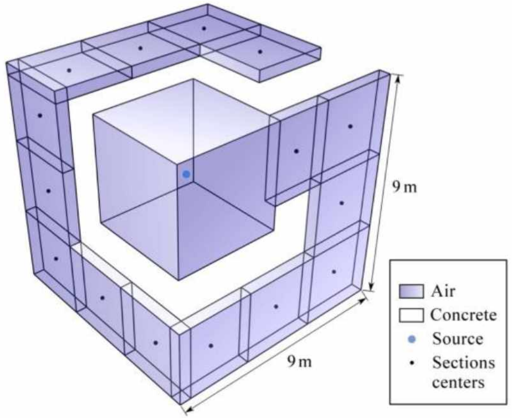
What are the benefits of such method on variance, clustering, … ?

What further applications of the method in reactor physics might be possible?

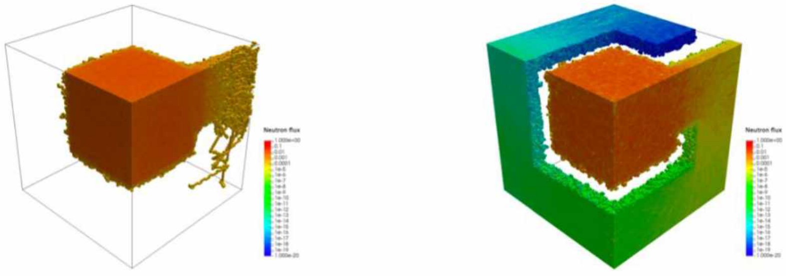
Adaptive Multilevel Splitting (AMS) – State of the art

- Implemented at CEA for variance reduction for radiation protection calculations [1]
- The idea is to attract particles towards a target (detector) by **ranking** and **re-sampling** particle histories
- The method is **robust, easy to use** and has shown good results in shielding !

Can we adapt it to criticality ?



From [1] : H. *Louvin (2017)*



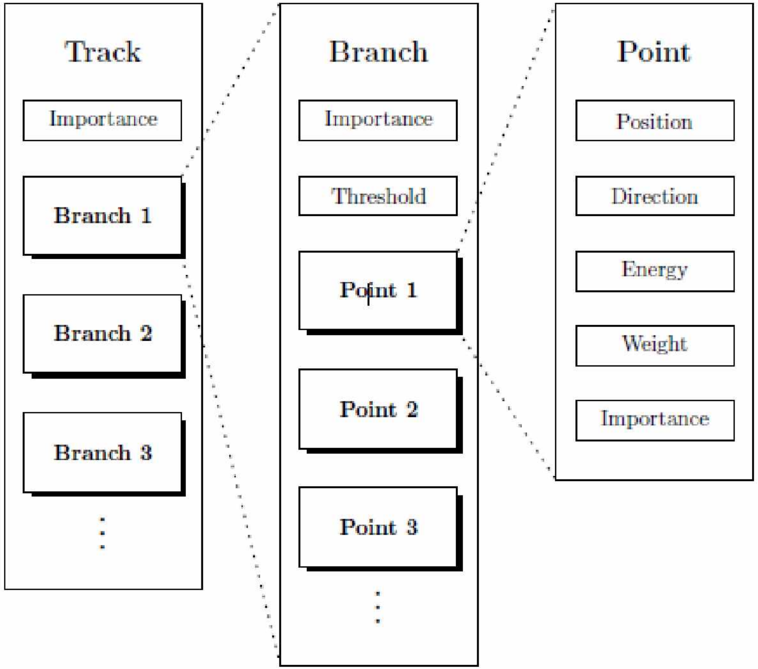
(a) Analog neutron flux (b) AMS neutron flux

Attenuation $\sim 10^{20}$ **!**

The AMS structure

- AMS track = neutron history
- AMS branch = one particle
- AMS point = collision, boundary crossing, …

Re-sampling tracks → re-sampling "independent" neutron histories

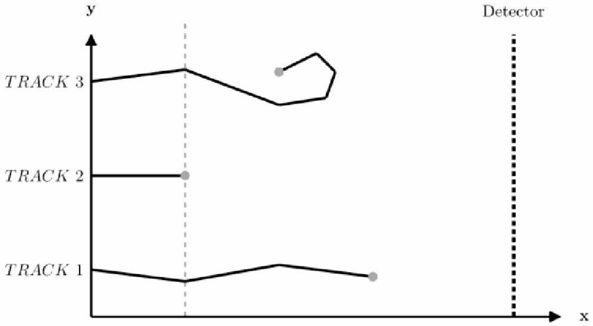


From [1] : H. *Louvin (2017)*

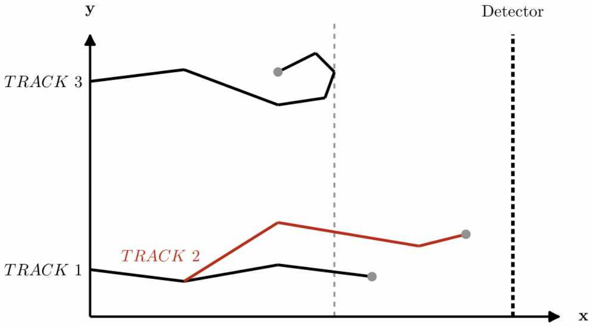
How does the AMS iterate

N = 3 tracks
K = 1 re-sampled at each iteration

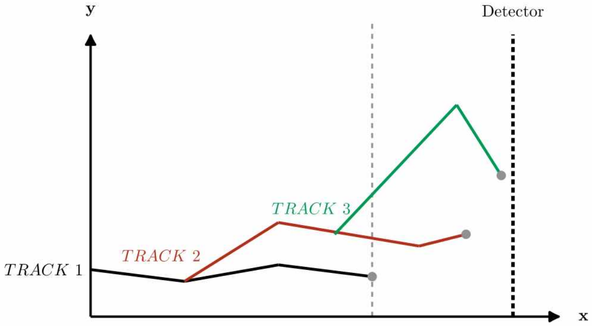
$$Importance \propto \frac{1}{distance\ to\ the\ detector}$$



Iteration 0 :
Analog transport of
all tracks



Iteration 1 :
Track 2 is deleted
and re-sampled

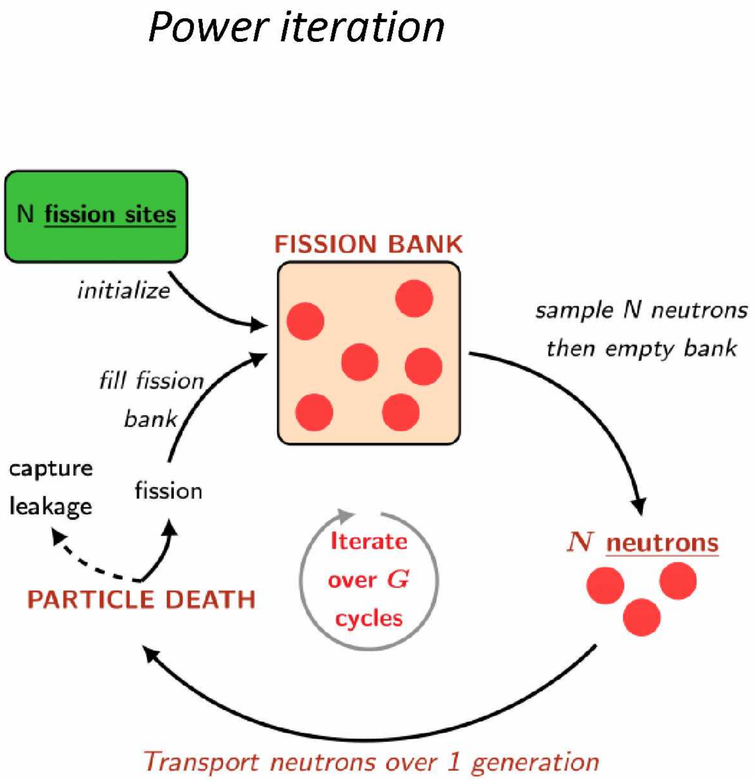


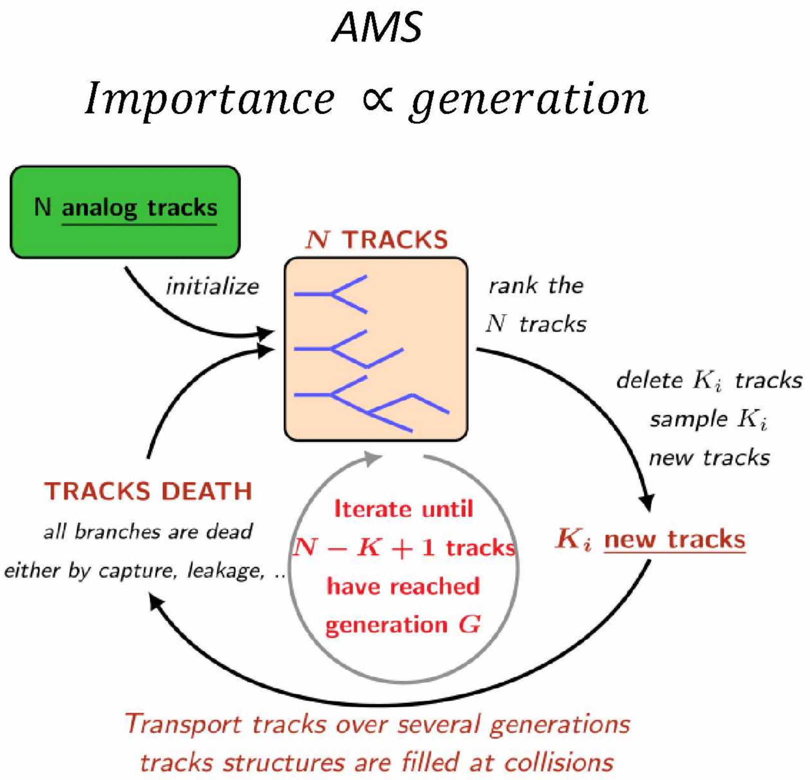
Iteration 2 :
Track 3 is deleted
and re-sampled

Final iteration : $\tilde{\phi} = \phi_{det} \times \alpha$

Probability to reach the detector : $\alpha = \left(1 - \frac{K}{N}\right)^{L}$

Application to criticality



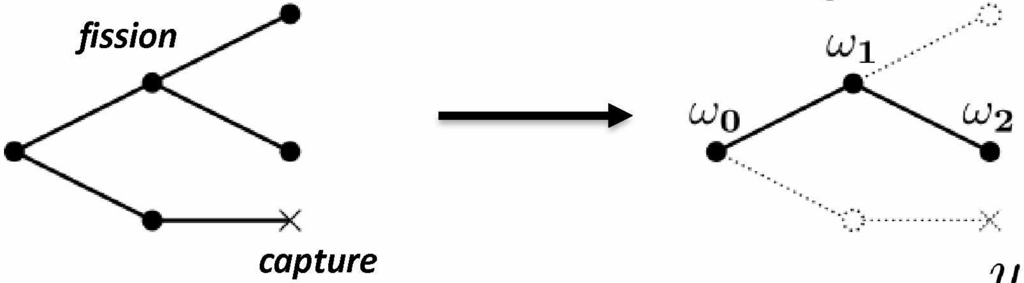


For AMS : criticality seen as an attenuation problem over successive generations

Handling multiplicative medium

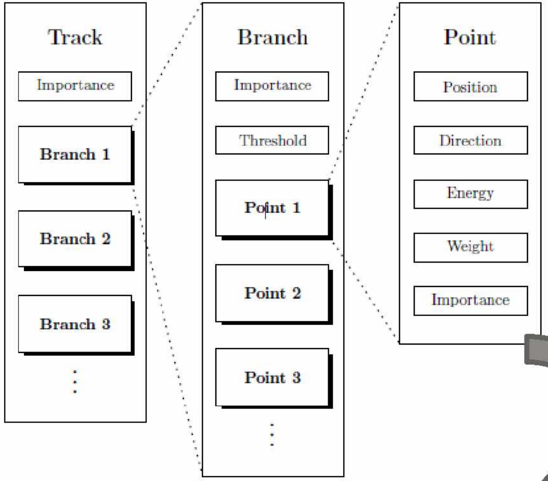
- Possible to handle branching tracks [1] , but…
- Increasing number of branches currently limits the use of AMS for **subcritical** systems
- We need a **numerically subcritical** system : ***branchless collision [2]***

Weight variations !



No more branches

$$w' = \frac{\overline{\nu}\Sigma_f}{\Sigma_a} w$$



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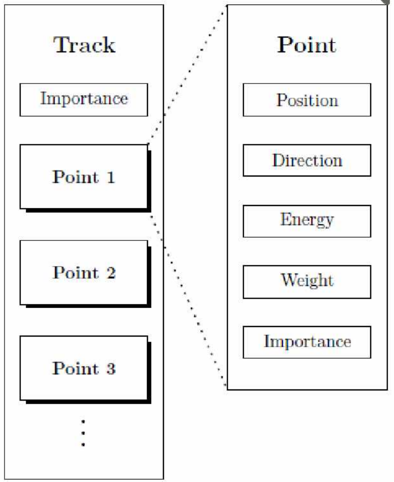


Application to a one-dimensionnal bare slab

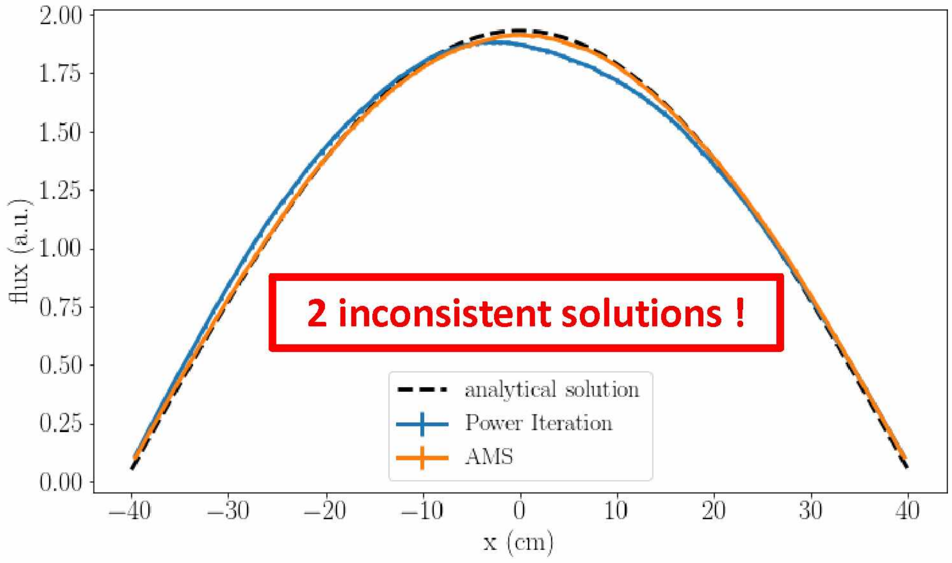
PROBLEM

- 80 cm One-dimensionnal bare slab
- Leakage
- One-speed problem
- 1000 neutrons/cycle, 1000 cycles, 100 independent runs
- Flux $\pm 3\sigma$
- AMS : $I = g + \frac{1}{1+|x|}$

Push neutrons through generations

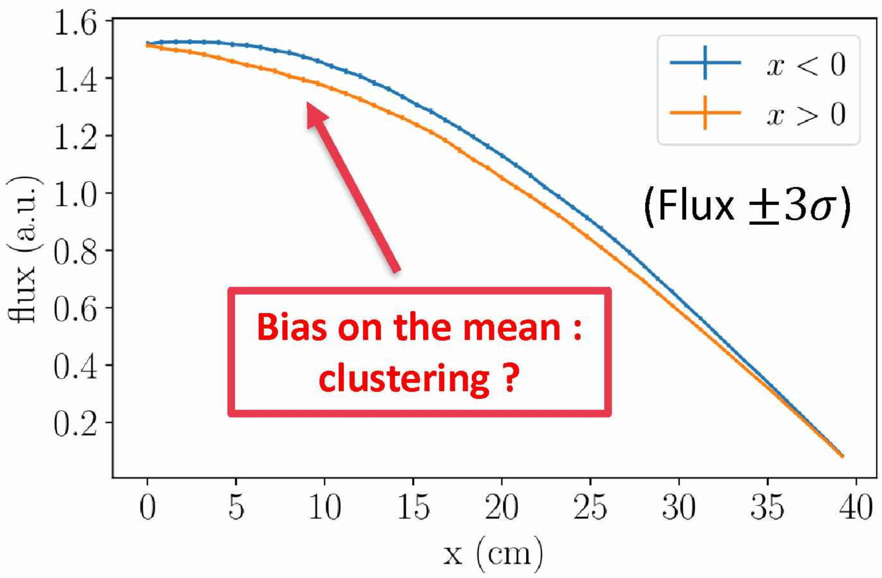
To avoid accretion in one generation

FLUX

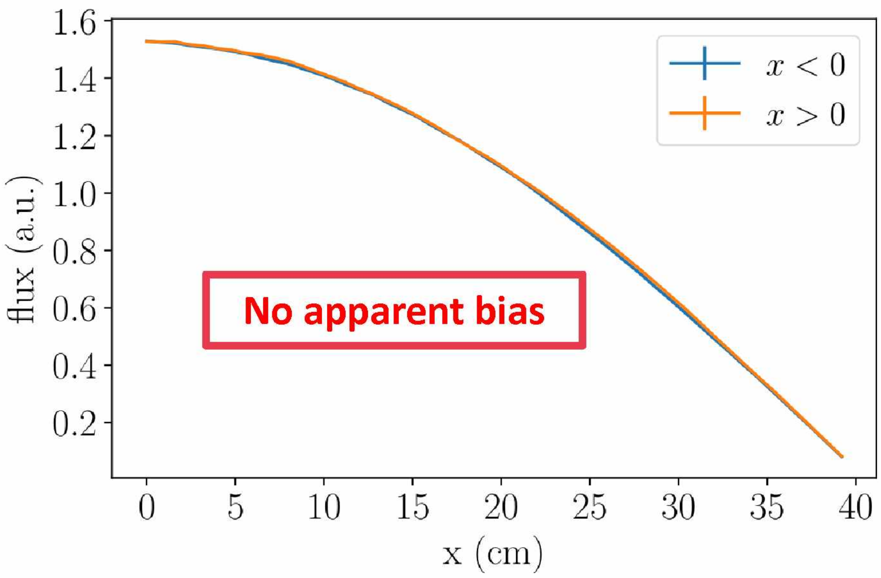


Homogeneous rod : symmetry of the solution

POWER ITERATION



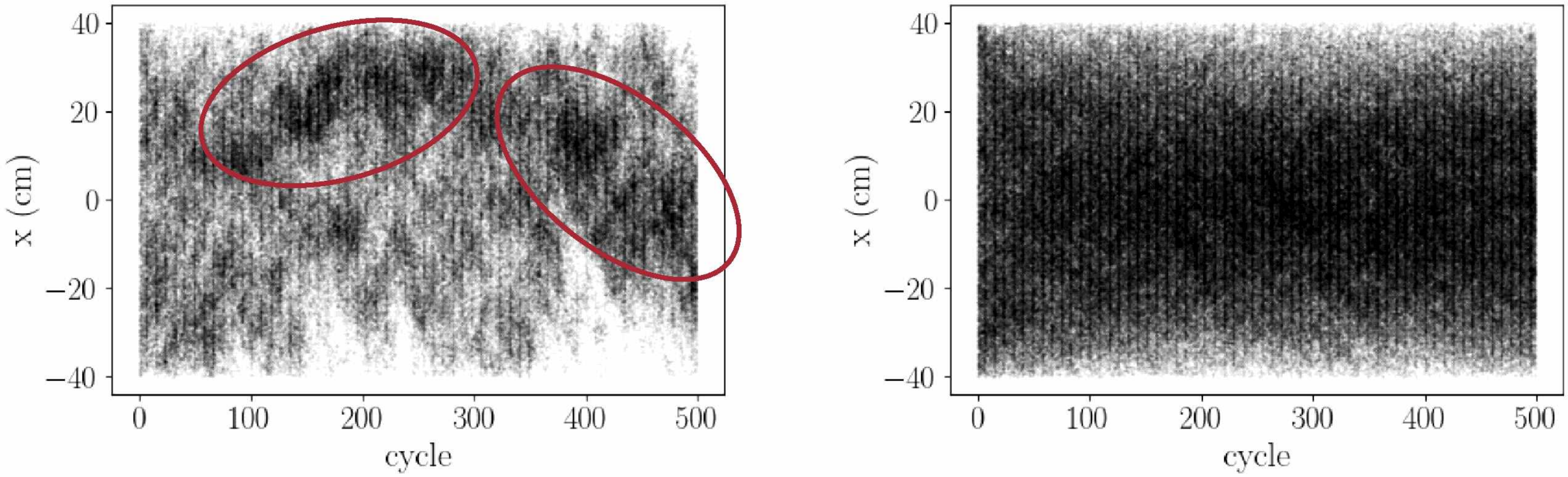
AMS + branchless



Formation of clusters in one simulation

Clusters are born from the imbalance between the birth (localized) and death (uniform) processes

POWER ITERATION **AMS + branchless**



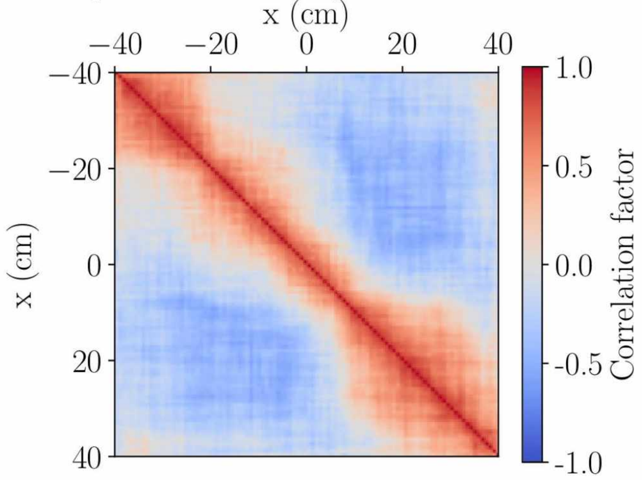
AMS + branchless : fewer visible clusters

Spatial correlations

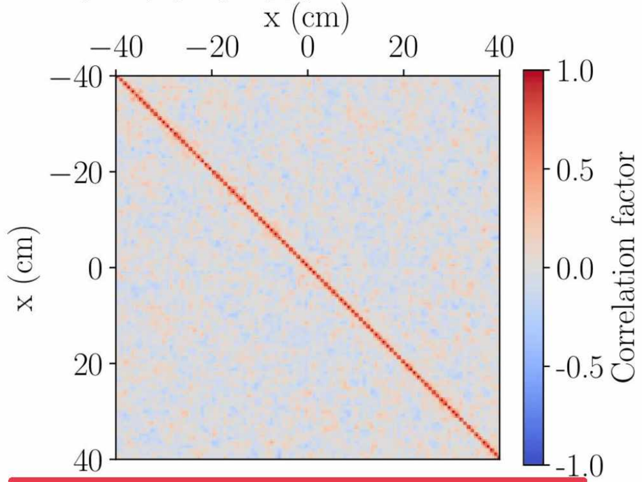
In one generation…

$$\rho_{i,j}(g) = \frac{Cov\left[\phi_g(x_i), \phi_g(x_j)\right]}{\sigma\left[\phi_g(x_i)\right]\sigma\left[\phi_g(x_j)\right]}$$

POWER ITERATION



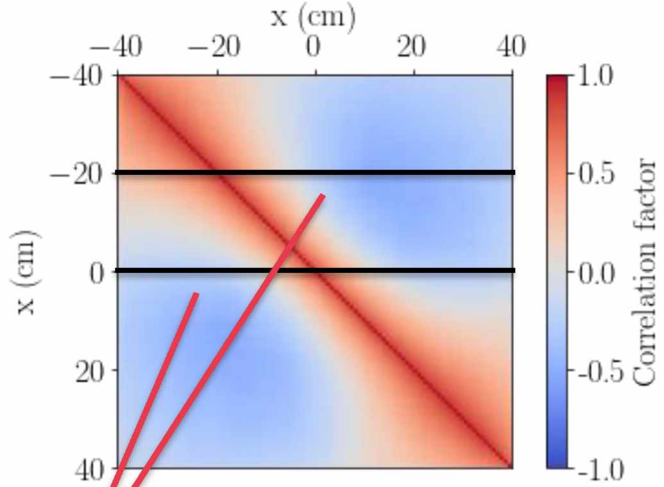
AMS + branchless



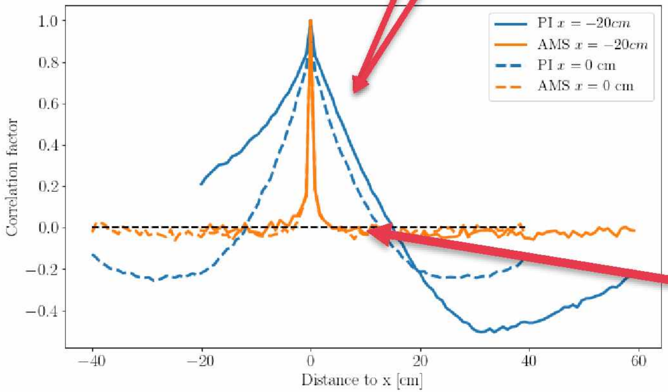
Spatial bins are barely correlated to each other !

Mean spatial correlations

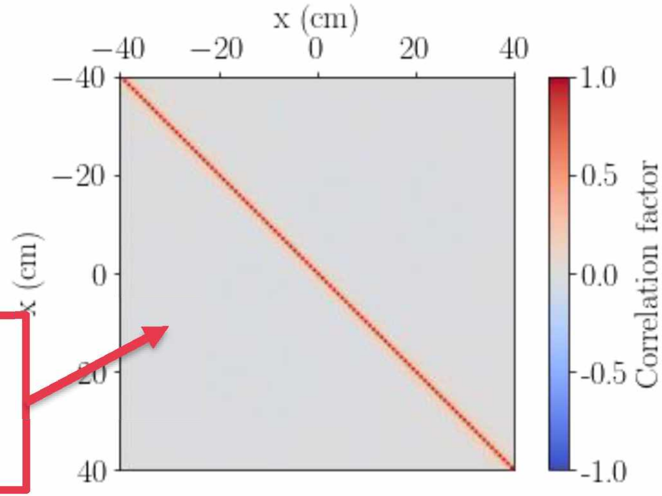
POWER ITERATION



$$\rho_{i,j} = \langle \rho_{i,j}(g) \rangle_g$$



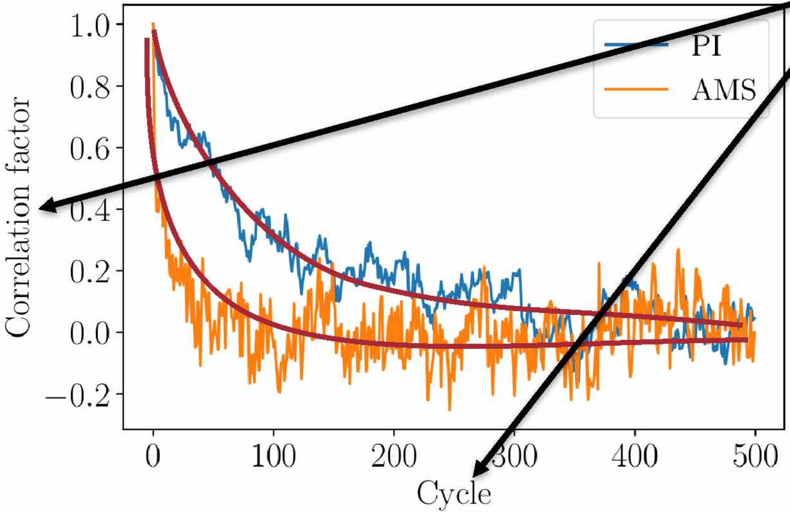
AMS + branchless

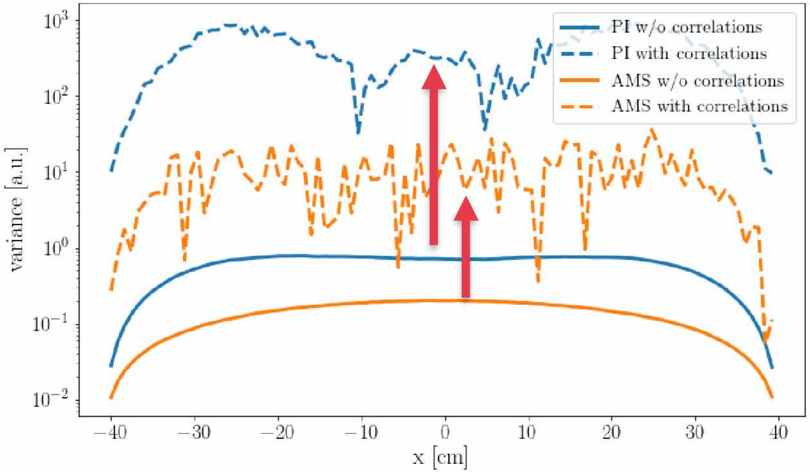


Less chances for clusters to appear and develop

Underestimation of the variance due to cycles correlations

Underestimation by a factor : $$1 + 2\sum_{k=1}^{n-1}\left(1-\frac{k}{n}\right)\rho_k$$





AMS + branchless : cycle-to-cycle correlations die more quickly

Real variance is less underestimated by AMS+branchless

Conclusion

Can we tweak the Adaptive Multilevel Splitting (shielding) to do criticality calculations ?

What are the benefits of such method on variance, clustering, … compared to the power iteration ?

Reduces bias on the flux (clustering due spatial correlations)

Reduces variance (observed and real – cycle correlations)

What further applications of the method in reactor physics might be possible?

Criticality (variance reduction, rod depletion [3,4], …)

Kinetics (population control and/or variance reduction)

Thank you for your attention

References

[1] Louvin, H. (2017). *Development of an adaptive variance reduction technique for Monte Carlo particle transport* (Doctoral dissertation, Université Paris-Saclay).

[2] Sjenitzer, B. L., & Hoogenboom, J. E. (2011). Variance reduction for fixed-source Monte Carlo calculations in multiplying systems by improving chain-length statistics. *Annals of Nuclear Energy*, *38*(10), 2195-2203.

[3] Cosgrove, P., Shwageraus, E., & Parks, G. T. (2020). Neutron clustering as a driver of Monte Carlo burn-up instability. *Annals of Nuclear Energy*, *137*, 106991.

[4] Brovchenko, M., Kodratoff, N., Importance of the Axial Burn-Up Modelling for Fuel Assembly Nozzles Activation Calculation, *PHYSOR 2022*

Appendix

How to rank the tracks ?

- Particle importance : probability to contribute to the score

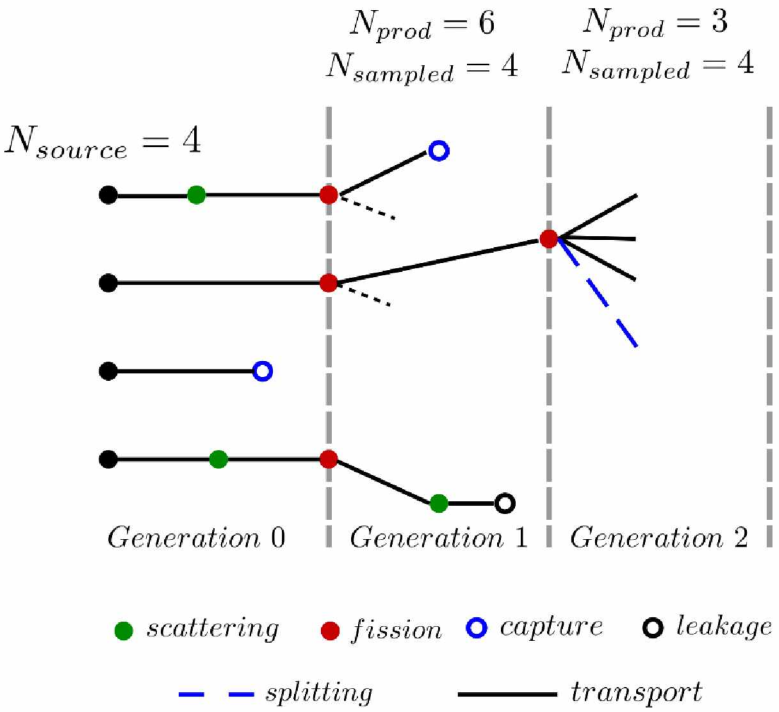
$$\textit{importance in } \vec{r}, E, \vec{\Omega} \textit{ at } t = \phi^*(\vec{r}, E, \vec{\Omega}, t)$$

- Problem : adjoint calculation at least as difficult as the forward problem

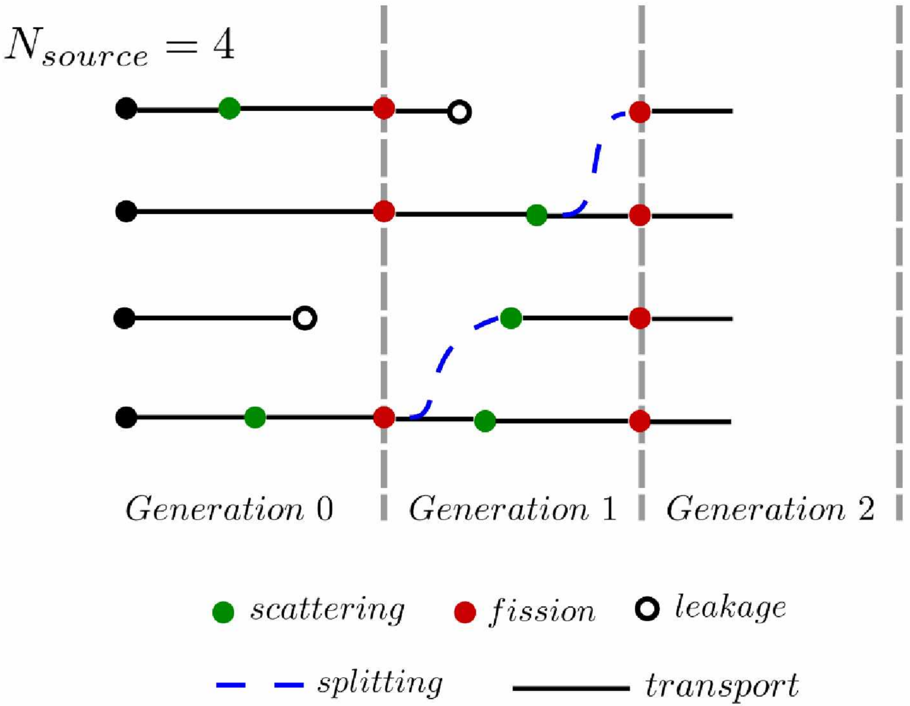
Even a rough estimation of ϕ^* could accelerate Monte Carlo !

DETERMINISTIC CALCULATION $\longrightarrow \phi^*(\vec{r}, E, \vec{\Omega}, t) \longrightarrow$ AMS $\longrightarrow$ **Variance reduction**

AMS + branchless VS Power Iteration



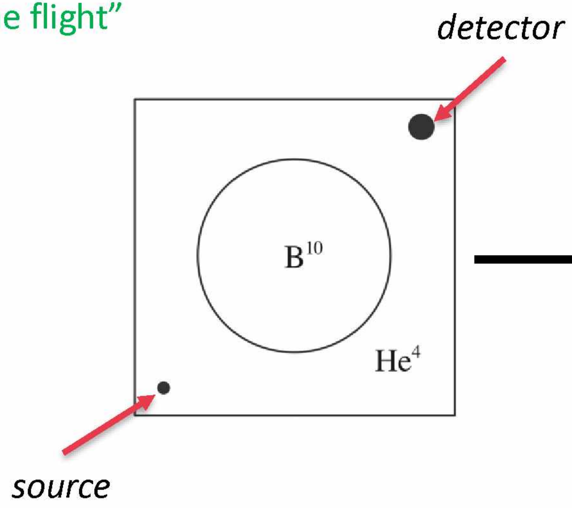
Power Iteration

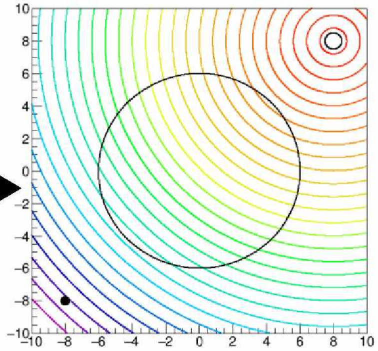


AMS with branchless collision

AMS for fixed source problems

- AMS for shielding applications
  - AMS = **Adaptative** Multilevel Splitting
  - Originates from applied mathematics (Cerou et al, 2007)
  - Applied to particle transport (Louvin et al, 2017)
  - Defines the splitting threshold “on the flight”
  - By using an importance map





Spatial importance map

$I \propto \frac{1}{r}$

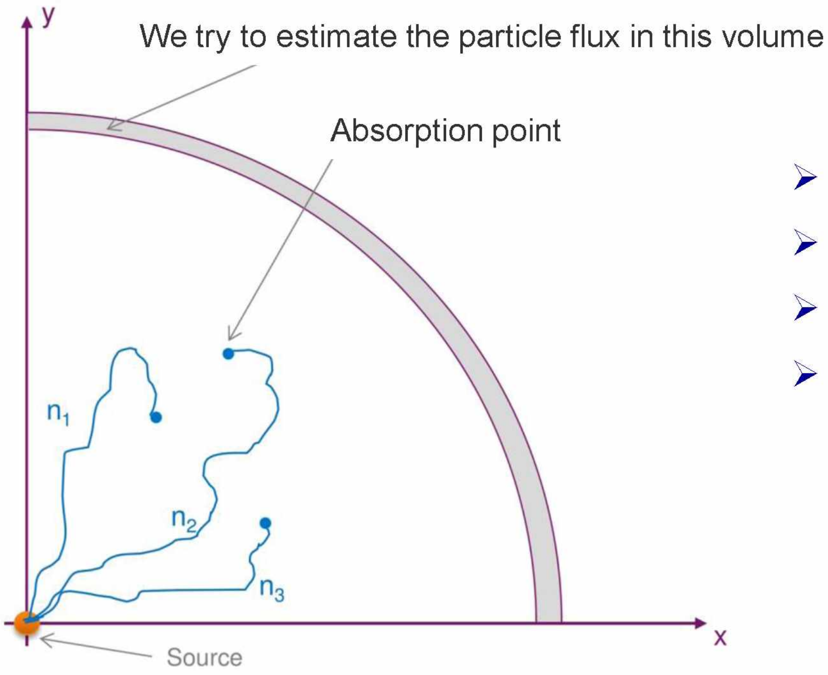
AMS for fixed source problems : the algorithm

- **Stopping criterium:**
  - **When n-k tracks have reached the "detector", the algorithm stops**
  - **The number of iteration corresponding to reach that criterium is N**
  - **Each neutron is assigned a statistical weight α being:**

$$\alpha = \left(1 - \frac{k}{n}\right)^{N}$$

- **An unbiased estimator of the flux is calculated "as usual" using the tracks of the last iteration**

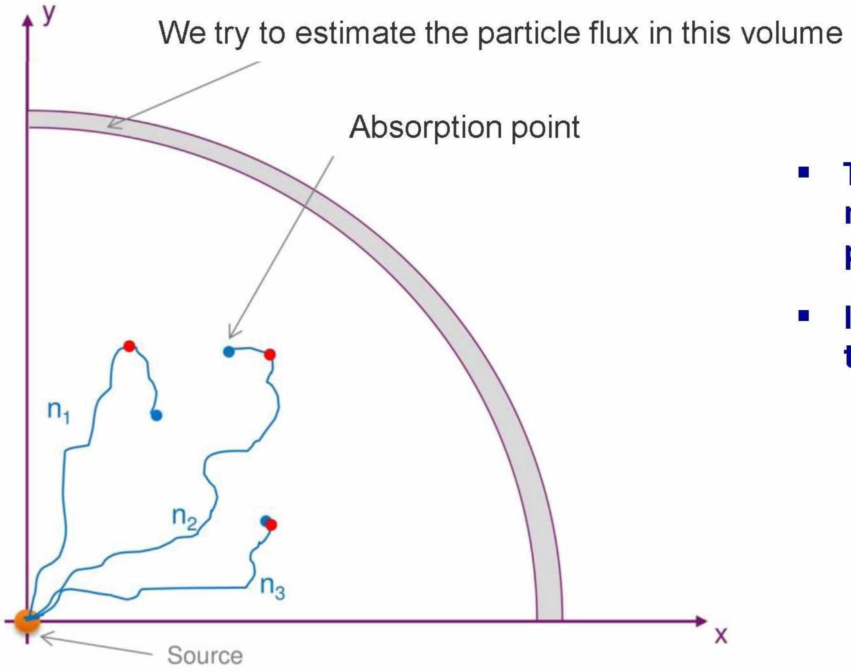
AMS for fixed source problems : the algorithm



- n=3 initial particles
- k=1 track resampled
- importance = distance to the source
- Target: spherical shell (purple)

H. Louvin, PhD thesis

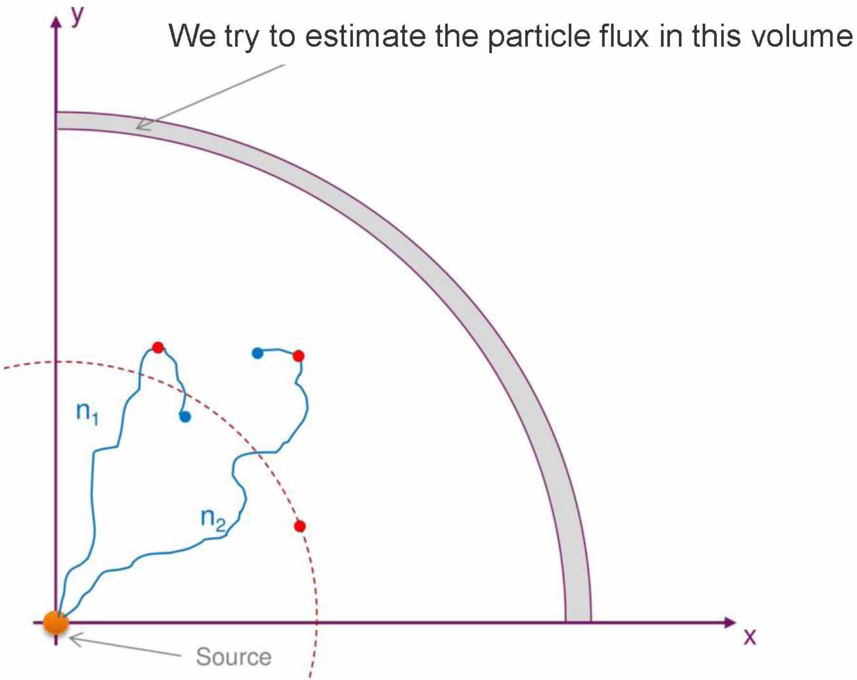
AMS for fixed source problems : the algorithm



- **The importance function is the maximal distance reached by the particle (red points)**
- **In this case the neutron tracks with the lowest score is n3**

H. Louvin, PhD thesis

AMS for fixed source problems : the algorithm

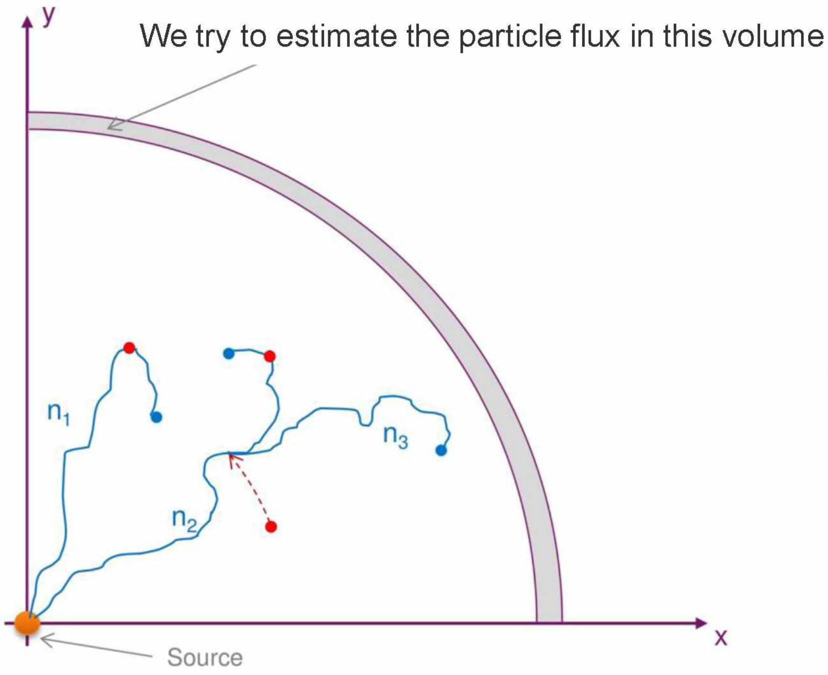


- **The tracks associated to particle 3 is deleted**

- **The maximum score of this track is stored and defines the first splitting level**

H. Louvin, PhD thesis

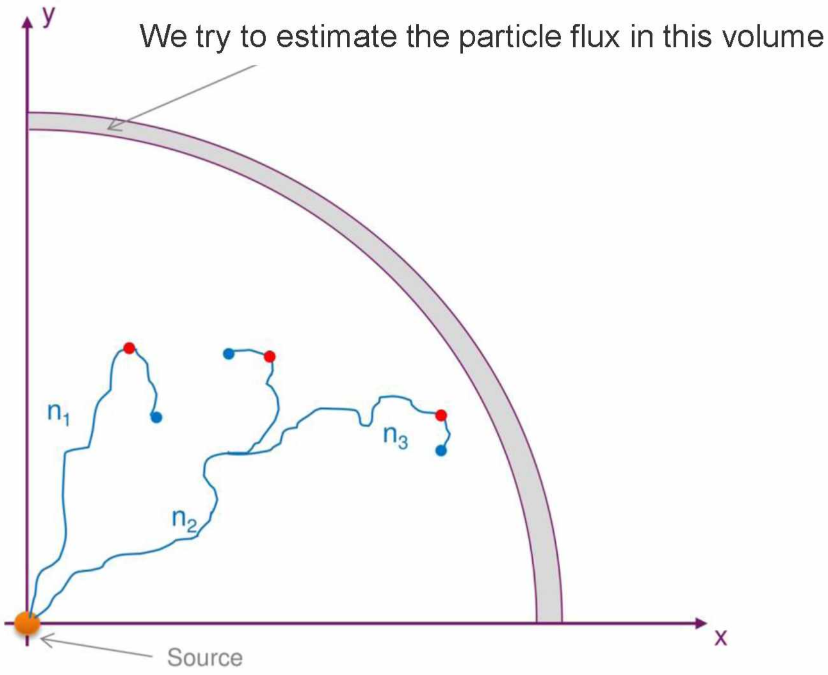
AMS for fixed source problems : the algorithm



- **Track number 2 is randomly sampled for the splitting**
- **A new particle is simulated from this splitting point until its absorption**

H. Louvin, PhD thesis

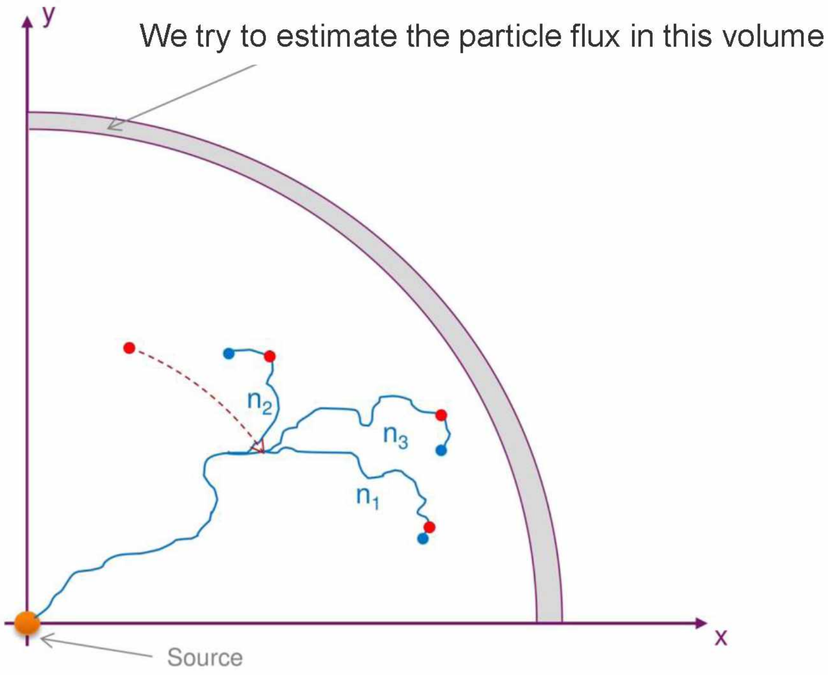
AMS for fixed source problems : the algorithm



- **The score of this new tracks n3 is calculated**
- **The first iteration is over**
- **The stopping criterium is not meet: the iteration process goes on**

H. Louvin, PhD thesis

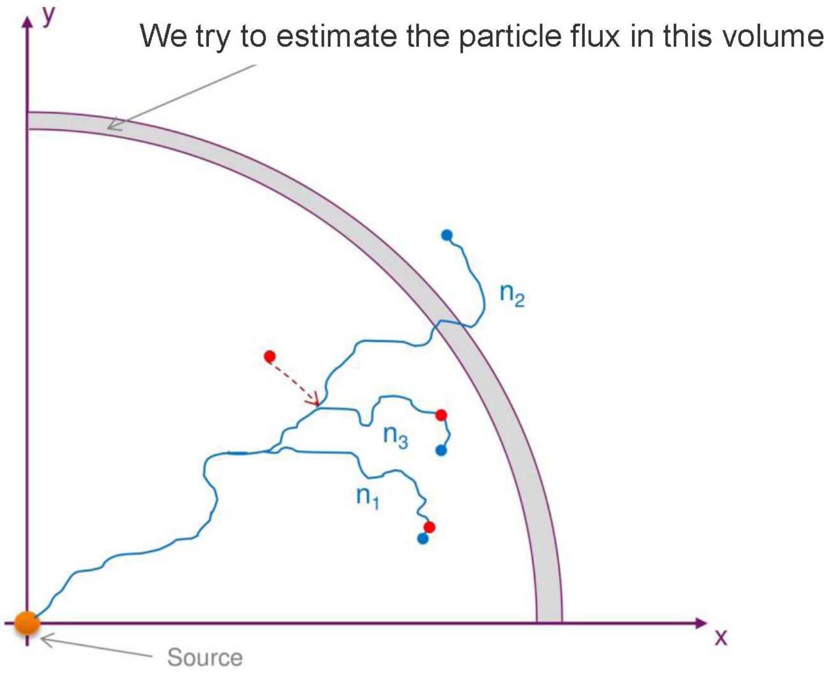
AMS for fixed source problems : the algorithm



- **Iteration 2**

H. Louvin, PhD thesis

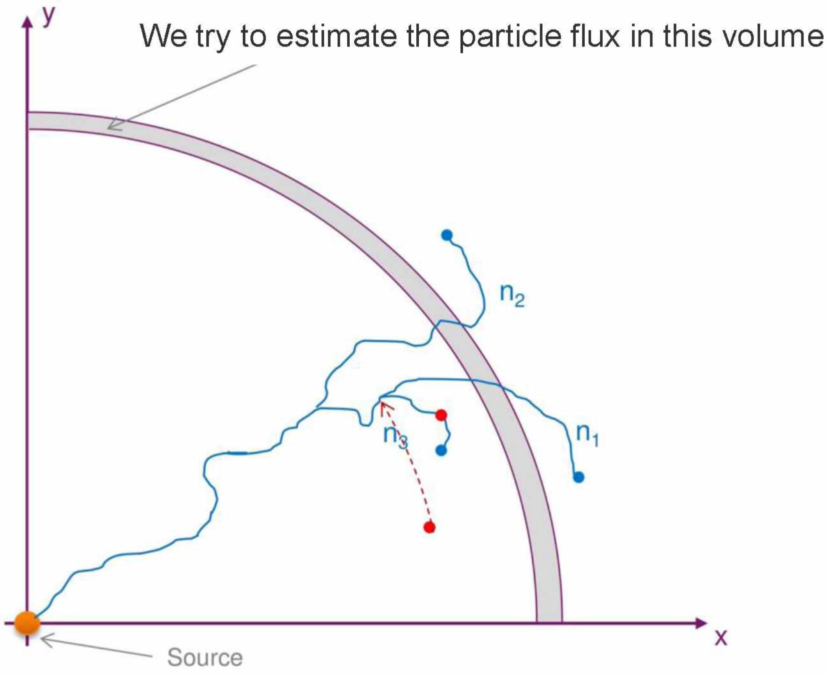
AMS for fixed source problems : the algorithm



- **Iteration 3**

H. Louvin, PhD thesis

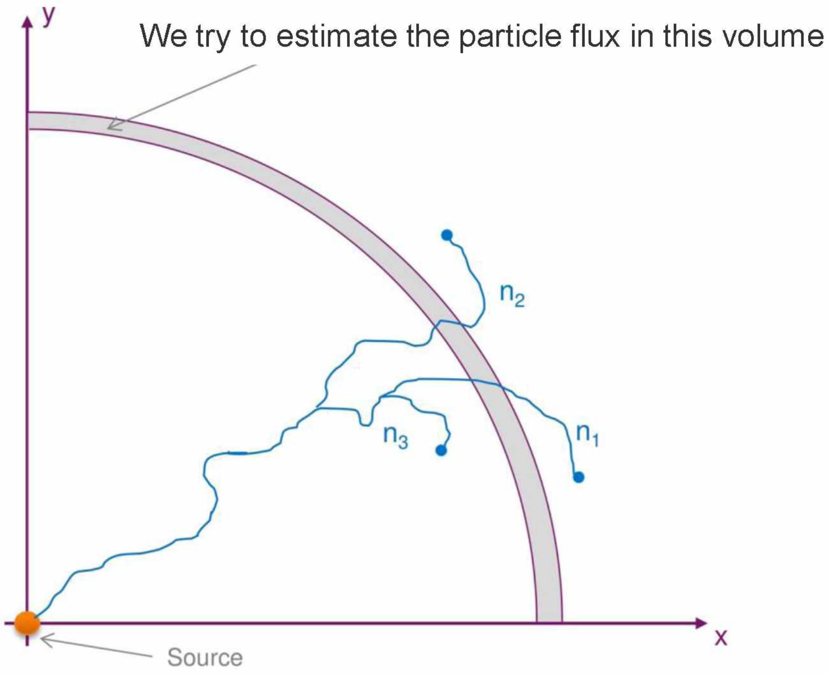
AMS for fixed source problems : the algorithm



- **Iteration 4**

H. Louvin, PhD thesis

AMS for fixed source problems : the algorithm



- **Iteration 4**
- **n-k particles have reached the target, the algorithm stops**
- **The statistical weight of all particles is :**

$$\alpha = \left(1 - \frac{1}{3}\right)^4$$

H. Louvin, PhD thesis