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Nuclear Data Uncertainties and Adjustments Using Deterministic and Monte-Carlo Methods along with PWR Measurements

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M&C 2023 - The International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering · Niagara Falls, Ontario, Canada · August 13 – 17, 2023



Introduction (1)

Introduction

Propagation of nuclear data uncertainties

PWR experimental detector responses

Nuclear data adjustment

Conclusions

(2)Propagation of nuclear data uncertainties

PWR experimental detector responses (3)

Nuclear data adjustement (4)

Conclusions (5)



Four fundamental sources of errors (*Neutron Physics*, P. REUSS, 2008) :

- (1) errors related to the simplified modelling of the physics
 - → models of resonance self-shielding effects,
 - → replace transport with diffusion, on full core level, etc.
- (2) errors related to imperfect numerical schemes applied to the obtained equations
 - discretizations, computer implementations
- (3) errors due to imperfect knowledge of nuclear data
- (4) errors in the system description \rightarrow dimensions, densities, isotopic compositions
 - → manufacturing tolerances

The **first two** errors constitute the **deterministic bias** (a systematic error), which can be computed as the **difference** with the Monte-Carlo method

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In the 1970s, when full core simulations were performed with 1-group diffusion :

→ "The albedo matrices are then often adjusted to experimental results. The albedo matrix elements are thus used as 'tuning' constants thay may vary, within reasonable limits, around their computed average values." (STAMM'LER and ABATTE, 1983)

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Later, fast diffusion coefficient of the reflector (D_1) is used :

→ "The reflector adjustment [...] reduces the discrepancies between the calculated and experimental detector responses in the peripheral assemblies at the beginning of the cycle by flattening the power distribution. [...] This adjustment corrects the discrepancy observed but does not constitute a definitive solution. The latter, currently under development, requires a reference calculation in one and two dimensional geometries." (our translation from KAMHA, 1981)





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→ "To fit calculations with respect to real in-core measurements, some reflector parameters can be adjusted. [...] Adjustment is done only on the reflector description by trying to decrease calculations-measurements differences. [...] Fictitious reflector [...] diffusion parameters (and mainly first group diffusion coefficient D₁ and absorption coefficient Σ_{a1}) can strongly be changed (some ten of percent) to fit in-core measurements." (ARGAUD, 1993)

- → "The fast diffusion coefficient of the radial reflector acts on the radial leaks and thus on the bulge of the radial flux distribution. It is adjusted to fit more accurately the experimental power output distribution over the core." (JOUTEL, 2015)
- → "The fast diffusion coefficient of the reflector has been set to 1.78 cm instead of 1.3 cm", i.e. a 37% increase, "on account of the significant experience of EDF in improving radial power distribution." (MARGUET, 2017)



- → Knowledge on sources of errors mentioned earlier \rightarrow unused
- → Instead, this assumption :
 - usual discrepancy between calculations and measurements, and
 - universal uncertainty of calculation
 - are taken as <mark>equal</mark> → numerous weaknesses
- → Uncertainty (of calculation) and discrepancy (between calculation and measurement) → concepts of radically different natures
- → Standard deviation (or dispersion) \neq uncertainty
- → Previously measured core, during a similar reloading → expecting similar discrepancy
- → Conversely, unmeasured cases may imply more discrepancy
 - → new reactors or fuel management, accident simulations

→ Fragilities accentuated for adjusted deterministic simulations (residuals)

➤ compensations between errors of different natures, unlikely to be universal

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Long-standing and still pending questions

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→ "What adjustments have been made to the TURTLE methodology to improve the agreement between calculation and measurement? How is the uncertainty in this adjustment accounted for in $F_{\Delta H}$, $F_{xy}(z)$ and $F_Q(z)$ uncertainties?" (NRC, 1987)

Recently renewed concerns

- → The French nuclear safety authority "is particularly attentive to feedback from the EPRs [...] in China, which highlights certain topics requiring specific investigations and instructions. This concerns, in particular, [...] anomalies in the power distribution in the core of the Taishan EPRs" (ASN, 2021).
- → To increase robustness → return to the fundamentals → sources of errors



Introduction

First start-up of Tihange-1, 900 MWe PWR



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A deterministic solution, with JEFF-3.3 Power distribution in Tihange-1, first start-up





Deterministic vs Monte-Carlo ↔ deterministic bias Power distribution in Tihange-1





Uncertainties on cross sections of hydrogen bound in H₂O Source : JEFF-3.3

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Propagation of hydrogen uncertainties through deterministic methods Total Monte-Carlo : 300 samples of nuclear data (with SANDY)

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Propagation of uncertainties from hydrogen, JEFF-3.3 Deterministic and Monte-Carlo comparison

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First start-ups of Bugey-2, Fessenheim-1 and 2, three identical 900 MWe reactors







Ranking of nuclear data uncertainties

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Simultaneous sampling : interactions between isotopes Detector responses

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	$\begin{array}{c} \mu{\pm}0.72 \\ \sigma{=}1.8\% \\ S{=}{-}0.0 \\ K{=}{-}0.0 \end{array}$	$ \begin{array}{c} \mu = 0.59 \\ \sigma = 1.7\% \\ S = -0.0 \\ K = -0.0 \end{array} $	$ \begin{array}{l} \mu : \text{ mean of the detector response} \\ \sigma : \text{ relative standard deviation (in \%)} \\ S : \text{ skewness (shown if } \sigma > 0.1\%) \\ K : \text{ excess kurtosis (if } \sigma > 0.1\%) \end{array} $							
	$\begin{array}{c} \mu = 1.24 \\ \sigma = 1.2\% \\ S = -0.1 \\ K = -0.0 \end{array}$	$\begin{array}{l} \mu = 0.84 \\ \sigma = 1.2\% \\ S = -0.1 \\ K = -0.0 \end{array}$	$\begin{array}{l} \mu {=} 0.82 \\ \sigma {=} 1.3\% \\ S {=} {-} 0.0 \\ K {=} {-} 0.0 \end{array}$	$\mu = 0.55$ $\sigma = 1.4\%$ S = 0.0 K = -0.1		Probability density				
	$\begin{array}{c} \mu = 1.05 \\ \sigma = 0.5\% \\ S = -0.1 \\ K = -0.0 \end{array}$	$\begin{array}{c} \mu = 1.32 \\ \sigma = 0.5\% \\ S = -0.1 \\ K = -0.0 \end{array}$	$\begin{array}{c} \mu = 0.94 \\ \sigma = 0.6\% \\ S = -0.1 \\ K = -0.0 \end{array}$	$\begin{array}{l} \mu = 0.77 \\ \sigma = 0.8\% \\ S = -0.0 \\ K = -0.0 \end{array}$	$\mu = 0.57$ $\sigma = 1.1\%$ S = 0.0 K = -0.1	0 Norr	.5 1.0 nalized det response	ector		
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Experimental detector responses during the 1st start-up of Bugey-2



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Experimental detector responses during the 1st start-up of Fessenheim-1



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Experimental detector responses during the 1st start-up of Fessenheim-2



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 \rightarrow Proximity between the measurement and a sample *k* of nuclear data estimated with

 σ_E

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$\chi_k^2 = \sum_{i=1}^{n_{\text{mes}}} \left(\frac{C_{i,k} - E_i}{\sigma_{E_i}}\right)^2$

→ Lack of uncertainty estimated by experimentalists
→ expert opinion, for lack of a better choice

$$=2\%$$
 (2)

→ Each nuclear data sample k → weight

$$w_k = \frac{\exp\left(-\chi_k^2/\chi_{\min}^2\right)}{\sum\limits_{\kappa=1}^{n} \exp\left(-\chi_\kappa^2/\chi_{\min}^2\right)}$$

(3)

(1)



Posterior detector responses (after BFMC)

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K = 0.0

0.5

1.0

	$\begin{array}{c} \mu = 0.72 \\ \sigma = 1.3\% \\ S = 0.0 \\ K = 0.0 \end{array}$	$\mu = 0.59$ $\sigma = 1.3\%$ S = 0.0 K = 0.0	μ : mean of the detector response σ : relative standard deviation (in %) S : skewness (shown if $\sigma > 0.1\%$) K : excess kurtosis (if $\sigma > 0.1\%$)						
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Modifications (red) to five prior quantiles (grey) of hydrogen cross sections, with BFMC

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4%2%(n,el)0 -2%-4% 10^{5} 10^{6} 10^{4} 10^{7} Energy [eV]



Discussion

- → D_1 reflector adjustment → depends on the reactor size, fuel loading, etc.
 - transferring information is rather difficult to justify
- → Nuclear data are universal, identical for all reactors
 - ➤ universal adjustment

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- → D_1 reflector adjustment → best estimate only, without any notion of uncertainties, must be estimated afterwards
- → Here, nuclear data adjustment also applies to uncertainties
- → Limit : relies on Monte-Carlo to compute and evict the deterministic bias



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→ For variations in nuclear data typically corresponding to their uncertainties :

Deterministic bias is constant, identical for \neq samples

- → Thus, it is possible to benefit from the "best of both worlds" through combination of
 - the **best estimate** from the **Monte-Carlo** method (mean) and
 - the uncertainties obtained with deterministic methods (moments of higher orders),
- → Second theme : adjustments. Objective : tend toward better extrapolability than adjustment on reflector D_1
- PWR experimental detector responses

Nuclear data adjustment

- → It seems legitimate to consider that computations should reproduce measurements made experimentally on real reactors. However, the standard adjustment for PWRs is very questionnable → attempt to improve as much as possible, to remove as much deficiencies as possible
- → Based on a physical knowledge of the underlying causes of errors → universality is sought instead of error compensation
- → Nevertheless, such an adjustment could still be questionable. In your opinion, what is the most convincing argument against such adjustments?



Open source software, open data

→ Totality of developed software and produced data (initial, intermediate and final) available on :

github.com/IRSN/SalinoPhD

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Examples :

- Nuclear data and its processing, from ENDF to ACE, Draglib
- Datasets for Serpent, Dragon and Donjon
- Experimental data (found in public sources) in csv, plots...
- → Such a principle of openness is intended to allow, in a pragmatic and effective way :
 - the verifiable and complete **reproducibility** of this publicly funded research
 - the transparency necessary for a rigorous peer review
 - wide dissemination of the developed ideas
 - **facilitate** interaction and collaboration



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Backup slides



Standard error on standard deviation (left axis), on skewness and on excess kurtosis (right axis)





Importance of power distribution

- → Most of the research in the field of uncertainty is focused on k_{eff} , yet, for reactors
 - nuclear safety depends not much on it
- → Focusing on power distribution
 - nuclear safety depends heavily on it (KERKAR and PAULIN, 2008)



C and D control rods banks in Tihange-1





Equivalence methods for the reflector

- \rightarrow Equivalences used for fuel \rightarrow not applicable to the reflector, as they typically conserve
 - diffusion properties (diffusion coefficients),
 - reaction rates (SPH)
- → Conserving reaction rates in the reflector or neutron scattering in the reflector is of little interest
- → Rather wishes to maintain its reflective properties against an external source of neutrons
- → Specific equivalences → determining cross sections and diffusion coefficients preserving reflective properties of a (transport) reference



→ Analytical solution of the current at the interface (x = 0)

$$\frac{J_1}{\phi_1} = \sqrt{D_1 \left(\Sigma_{a1} + \Sigma_{1 \to 2} \right)} \tag{1}$$

and

$$\frac{J_2}{\phi_1} = \frac{\phi_2}{\phi_1} \sqrt{D_2 \Sigma_{a2}} - \frac{\Sigma_{1 \to 2} \sqrt{D_1 D_2}}{\sqrt{\Sigma_{a2} D_1} + \sqrt{(\Sigma_{a1} + \Sigma_{1 \to 2}) D_2}}$$
(2)

- → After analytical study, numerical experiments with a S_N transport code on a representative heterogeneous traverse
- → Reflector applicable for all fuel bundles, \forall temperature, burnup, etc.
 - → reflector response for fuel varying as such
- 1) Fast group $\rightarrow J_1/\phi_1$ is constant

$$R_1 = \frac{J_1}{\phi_1} \tag{3}$$



Lefebvre-Lebigot method (2/2)

- 2) Thermal group
 - $\rightarrow J_2/\phi_1$ depends on the fuel

$$\frac{J_2}{\phi_1} = -R_3 + R_2 \frac{\phi_2}{\phi_1} \tag{4}$$

→ 5 unknowns and 3 parameters (R_1 , R_2 et R_3) calculated in S_N , Lefebvre and Lebigot suggest to equalize D_g :

$$D_{1}^{\text{fuel}} \approx 1.3 \,\text{cm} \qquad D_{1}^{\text{reflector}} = 1.3 \,\text{cm} \\ D_{2}^{\text{fuel}} \approx 0.4 \,\text{cm} \qquad D_{2}^{\text{reflector}} = 0.4 \,\text{cm} \\ \Sigma_{a2} = \frac{(R_{2})^{2}}{D_{2}} \qquad \Sigma_{1 \rightarrow 2} = R_{3} \left(\frac{R_{1}}{D_{1}} + \sqrt{\frac{\Sigma_{a2}}{D_{2}}} \right)$$

$$\Sigma_{a1} = \frac{(R_1)^2}{D_1} - \Sigma_{1 \to 2}$$
(7)

Case B

(5)

(6)

<u>Slope</u> = R_2

Case A

*J*₂

 ϕ_1

 $-R_3$

0



k_{∞} discrepancy between Serpent and Dragon (pcm), for assemblies during 1st start-up of Tihange-1









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Posterior (BFMC) correlations between the uncertainties of hydrogen and uranium 238





Prior correlations between the uncertainties of diffrent isotopes



Posterior (BFMC) correlations between the uncertainties of different isotopes





- 1) JEFF-3.3 or ENDF-B/VIII.0 evaluations → covariance matrices
- → Multigroups \rightarrow piecewise constant
- → Sometimes missing or incomplete (minor isotopes)
- 2) TENDL-2019 evaluation \rightarrow some various benefits
- → Very complete uncertainties (angular distributions, etc), continuously varying, not necessarily Gaussian
- → Above all, single code for production \rightarrow high format homogeneity \rightarrow safer and faster
- → TALYS optical models → no evaluation for less than 20 nucleons (hydrogen, boron, oxygen...) → JEFF-3.3
- → No evaluation either for some heavy isotopes, the most important ones : uranium 235, 238... → JEFF-3.3

The idea is not to demonstrate the qualities or defects of JEFF-3.3 or TENDL-2019 : just a choice



Propagation of uncertainties from uranium 238, JEFF-3.3 300 samples of nuclear data (with SANDY)





Propagation of uncertainties from uranium 238, JEFF-3.3 Deterministic and Monte-Carlo comparison





Hydrogen uncertainty decomposition with respect to reaction and energy



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Uranium 238 uncertainty decomposition with respect to reaction and energy From Luka Stancev's internship





Drakkar detector responses (JEFF-3.3)



Discrepancy between Drakkar (JEFF-3.3) and experimental mean





Power distribution discrepancy between Drakkar and Serpent, on Bugey-2 and Fessenheim-1 and 2

Ī									г 10.0%
1	1.2	0.2		Dra	-75%				
2	2.6	2.0	1.4	1.1					1.070
3	1.2	1.3	1.5	1.6	0.1				- 5.0%
4	-0.1	-0.2	0.2	0.6	0.9	0.1			- 2.5%
5	-1.8	-1.2	-1.0	-0.2	0.6	1.6	1.1		- 0.0%
6	-2.4	-2.4	-1.6	-1.0	0.2	1.5	1.4		2.5%
7	25	2.0	2 · 3	1 9	0.2	1 2	2.0	0.2	5.0%
1	-0.0	-2.9	-2.4	-1.2	-0.2	1.0	2.0	0.2	7.5%
8	-3.4	-3.5	-2.4	-1.8	-0.1	1.2	2.6	1.2	-10.0%
	Η	G	\mathbf{F}	Ε	D	С	В	А	10.070



Discrepancies between Drakkar (corrected from deterministic bias, JEFF-3.3) and experimental mean





Simultaneous sampling : interactions between isotopes Power distribution





 \rightarrow Proximity between the measurement and a sample *k* of nuclear data estimated with

$$\chi_k^2 = \sum_{i=1}^{n_{\text{mes}}} \left(\frac{C_{i,k} - E_i}{\sigma_{E_i}} \right)^2$$

→ Lack of uncertainty estimated by experimentalists
→ expert opinion, for lack of a better choice

$$\sigma_E = 2\% \tag{9}$$

→ Each nuclear data sample k → weight

$$w_{k} = \frac{L(\mathbf{p}_{k}, \mathbf{x})}{\sum_{\kappa=1}^{n} L(\mathbf{p}_{\kappa}, \mathbf{x})} = \frac{\exp\left(-\chi_{k}^{2}/2\right)}{\sum_{\kappa=1}^{n} \exp\left(-\chi_{\kappa}^{2}/2\right)}$$
(10)

where $L(\mathbf{p}_k, \mathbf{x}) \rightarrow$ likelihood of the **x** experimental observations for a *k* sample of **p** nuclear data

(8)













w weight histogram obtained with BMC





Posterior (BMC) detector responses



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Overfitting illustration, in green



Credits: I. ICKE, Creative Commons BY-SA 4.0 https://commons.wikimedia.org/wiki/File:Overfitting.svg



Modifications (red) to five prior quantiles (grey) of hydrogen cross sections, with BMC





Modifications (red) to five prior quantiles (grey) of uranium 238 cross sections, with BFMC





Modifications (red) to five prior quantiles (grey) of uranium 238 cross sections, with BMC





Modifications (red) to five prior quantiles (grey) of uranium 235 cross sections, with BMC





Modifications (red) to five prior quantiles (grey) of nickel 58 cross sections, with BMC





Modifications (red) to five prior quantiles (grey) of zirconium 90 cross sections, with BMC





Modifications (red) to five prior quantiles (grey) of zirconium 90 cross sections, with BFMC





Pseudorandom number generator

- → "The generation of random numbers is too important to be left to chance" (COVEYOU)
- → Codes such as **SERPENT** (since version 2.1.0) and MCNP5
 - → Linear congruential generator

$$X_{n+1} = (a \times X_n + c) \mod m$$

→ Nuclear data sampling
$$\rightarrow$$
 SANDY \rightarrow NumPy

- → Since NumPy version 1.17, permuted congruential generator (PGC64)
- → Before NumPy 1.17, Mersenne Twister
- → Random selection of nuclear data
 - → Here, relies on Python 3.6.9 → Mersenne Twister

(11)