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Jean-Sébastien Affagard, Baptiste Louis, Alain Gerard, Florence Gauthier, Anne Cecile Jouve. A simplified mechanical system to determine the delayed impact of the content on the LID. The International Symposium on the Packaging and Transportation of Radioactive Materials, PATRAM 22, Jun 2023, Antibes (France), France. irsn-04201652

HAL Id: irsn-04201652

<https://irsn.hal.science/irsn-04201652v1>

Submitted on 11 Sep 2023

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11-15 June 2023, Juan-les-Pins, France

A SIMPLIFIED MECHANICAL SYSTEM TO DETERMINE THE DELAYED IMPACT OF THE CONTENT ON THE LID

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ABSTRACT

For approved packages, the IAEA regulations require tests to simulate accident conditions to ensure that the consequences of such events are limited. In particular a 9-meter drop test must be performed. In case there is a significant gap between the contents and the lid, the kinematics of the impact could lead to a delayed impact of the content on the lid. So, the kinematic energy transmitted to the lid could be non-negligible due to the stiffness of the impact, thus leading to a strain of the screws which could no longer guarantee the safety performance of the package compared to a configuration with no gap.

As the technical support organization of the French Nuclear Authority (ASN), IRSN is developing a tool to quickly evaluate whether complex, time-consuming and costly experimental tests or numerical analyses are required to evaluate the consequences of the delayed impact phenomenon on the safety level of the package design.

This tool uses a simplified model based on Signorini's conditions. The content is modelled as beams discretized by finite elements and the contact is managed by a Lagrange multiplier to better represent the physical phenomena, at least compared to a simple mass/spring model, notably the contact time and the deceleration shape/amplitude. After demonstrating the validity of the model parameters, they will be adjusted through comparisons with actual drop tests that simulate the delayed impact. Then sensitivity analyses will be carried out to apprehend the influence of the damping, the incompressibility of the content, the non-linearity of the behaviour, etc.

INTRODUCTION

For approved packages, the IAEA regulations [1] require tests to simulate accident conditions to ensure that the consequences of such events are limited. In particular, a 9-meter drop test must be performed. The tests must be conducted under the most unfavorable conditions from the point of view of criteria such as activity release, containment and radioprotection. During an axial drop of the package, the lid/content interaction may be critical if the content is free, resulting in a hard impact without much damping. Gaps between the content and the lid or package walls exist because of their geometrical dimensions and the possible effect of differential expansion between these components. If the content is not fixed, its relative movement in the cavity can affect the dynamic behaviour of the package and extremely raise the impact loads on the lid system. Obviously, the loads acting on the components of the content in these internal collisions can be significant. The reasons for the occurrence of the internal impacts in drop tests with packages having mobile contents are discussed in papers by BAM [2, 3, 4, 5]. Thus, in case where a significant gap is present between the content and the lid, the shock absorber will impact the ground first then the content will impact the lid with a

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delay [6], which induces a difference in speed between those two elements. Therefore, the kinematic energy transmitted to the lid could be non-negligible due to the stiffness of the impact, thus leading to a strain level in the screws of the lid which could no longer guarantee the safety performance of the package compared to a configuration with no gap. Experimental tests or numerical analyses, which can be complex, time-consuming and costly, are then required to evaluate the consequences of the delayed impact phenomenon on the safety level of the package design.

In the literature, several studies consider the interaction between the cask and its content either numerically [7, 8, 9] or analytically [10, 11]. Finite element modeling allows performing parametric simulations. For example, the loading conditions [7] and the effect of types of contents [8] were investigated. More recently, a numerical analysis with a delayed impact was carried out on the TS-69 cask [9]. This paper concludes that the higher the initial gap, the higher the acceleration on the lid.

In general, finite element modelling of a complete package model is rather long and expensive. As a technical support organization (TSO), IRSN assesses package design safety report upon request by the French nuclear safety authority (ASN). In this framework, faster modelling to help assessing the mechanical performance of packages is necessary. In [10] and [11], simple mass-spring analytical models were developed. In [10], the relationship between cask and content accelerations in drop tests for several parameters was examined [10], while in [11] the effect of axial gap on the impact response of a cask lid was estimated. In addition, [12] showed a more complex model compared with experimental data.

To complete the work presented above, IRSN is developing their own tool to quickly evaluate whether experimental tests or numerical analyses are required to evaluate the consequences of the delayed impact phenomenon on the safety level of the package design. This tool will be used to clarify how delayed impact effects can affect the package performance, especially by identifying which gap size likely to lead to the disadvantageous effects. IRSN decided to use a simplified model based on Signorini's conditions [13] where the content is modelled as beams discretized by finite elements, and the contact is managed by a Lagrange integrator to better represent the physical phenomena, at least compared to a simple mass/spring model, notably the contact time and the deceleration shape/amplitude [14, 15, 16].

MATERIAL AND METHODS

The objective of this section is to present how contact is designed in the current IRSN model. Contrary to many finite element softwares, which impose a "penalty" during contact, the model presented in this paper will impose the absence of penetration through a variational Lagrange integrator. The main disadvantage of the penalty method is that the solution depends on the choice of the penalty coefficient. The method of Lagrange integrator can overcome this problem by introducing a parameter with a physical meaning. However, this increases the size of the problem thus the computation time, which is fortunately less significant in the simplified model developed at IRSN. The following equations summarize the formulation presented in [14, 15, 16] for explicit contact dynamics, called "CD-Lagrange time integrator". If we consider two deformable bodies, on the body boundaries, three distinct fields can be distinguished: Dirichlet conditions, Neumann conditions and contact interface between the two bodies. The contact conditions called HSM conditions (Hertz – Signorini – Moreau [13]) for a frictionless contact, under the small displacement assumption can be written as:

$$g_N = |(X_2 + u_2) - (X_1 + u_1)| \cdot n_1 \geq 0 \quad (1a)$$

$$\tau_N = \underline{\sigma}_k \cdot n_k \cdot n_k \leq 0 \quad , k = 1,2 \quad (1b)$$

$$g_N \cdot \tau_N = 0 \quad (1c)$$

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with \underline{X}_k , a pair of contact points belonging the contact interface, g_N the gap between the two points of the contact interface and \underline{u}_k the displacements of the two contact points. The normal stress is represented by τ_N , $\underline{\sigma}_k$ is the stress tensor at the contact points and \underline{n}_k the normal to the surface at the contact points.

The equation (1a) represents the condition of impenetrability. This gap function means the two bodies can be separated ($g_N > 0$) or contiguous without penetration ($g_N = 0$). The equation (1b) imposes that the normal contact stress (τ_N), always negative, is either inactive ($\tau_N = 0$) or active ($\tau_N < 0$). The third condition (eq. 1c) describes the fact that there is compression only when the bodies are in contact ($g_N = 0$ and $\tau_N < 0$). Otherwise the work done by the contact forces is zero ($g_N > 0$ and $\tau_N = 0$). According to Moreau [17], the contact conditions can be rewritten through a velocity-impulse as:

$$\begin{aligned} \text{if } g_N > 0, & \quad \text{then} & \quad i_N = 0 \\ \text{if } g_N = 0, & \quad \text{then} & \quad \begin{cases} \dot{g}_N \geq 0 \\ i_N \leq 0 \\ \dot{g}_N \cdot i_N = 0 \end{cases} \end{aligned} \quad (2)$$

with i_N the contact impulse, \dot{g}_N the normal component of the relative velocity between the two contact points. The gap velocity is defined as:

$$\dot{g}_N = (\underline{\dot{u}}_2 - \underline{\dot{u}}_1) \cdot \underline{n}_1 \quad (3)$$

with $\underline{\dot{u}}_n$ the velocity of the contact point n .

In the IRSN model, a spatial finite element discretization is used for each deformable body. Thus, according to the explicit central difference scheme on the step time ($\Delta t = [t_n; t_{n+1}]$), the problem can be written as a discrete equation in terms of space and time as:

$$\begin{cases} \underline{M} \underline{\ddot{U}}_{n+1} + \underline{C} \underline{\dot{U}}_{n+\frac{1}{2}} + \underline{K} \underline{U}_{n+1} = \underline{f}_{ext, n+1} + \underline{f}_{cont, n+1} \\ + \text{HSM contact conditions} \end{cases} \quad (4)$$

with \underline{M} the lumped mass matrix, \underline{C} the damping matrix, \underline{K} the stiffness matrix, $\underline{f}_{ext, n+1}$ the external forces, $\underline{f}_{cont, n+1}$ the contact forces and \underline{U}_{n+1} , $\underline{\dot{U}}_{n+\frac{1}{2}}$ and $\underline{\ddot{U}}_{n+1}$ the displacements, velocities and accelerations for bodies.

From the equation (4), Fekak et al. [14] proposed to handle the contact to add the impulse of the nodes in contact at time $n + 1$ from a restriction operator identifying the degrees of freedom involved in the contact in the direction normal to the interface at that time, noted by $L_{N,n+1}$ and the Lagrange multiplier (λ_N) defined at the time $n + \frac{3}{2}$ as:

$$\underline{L}_{n+1} = (\underline{L}_{N,n+1})^T \underline{\lambda}_{N,n+\frac{3}{2}} \quad (5)$$

Thus, written at time $t_{n+\frac{3}{2}}$, by using the step time Δt and the central difference approximation for accelerations, the finite element discretization becomes:

$$\underline{M} \underline{\dot{U}}_{n+\frac{3}{2}} = \underline{M} \underline{\dot{U}}_{n+\frac{1}{2}} + \Delta t \left(\underline{f}_{ext, n+1} - \underline{C} \underline{\dot{U}}_{n+\frac{1}{2}} - \underline{K} \underline{U}_{n+1} \right) + \underline{L}_{n+1} \quad (6)$$

Finally, multiplying, both sides of the previous equation by $(\underline{L}_{N,n+1} \underline{M}^{-1})$, gives the local impact/contact equation to be solved. The energy balance can be written with ΔW_{kin} , ΔW_{comp} , ΔW_{ext} , ΔW_{int} , ΔW_{IC} ,

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respectively the increments over the time step of the kinetic, internal, complementary, external and impact/contact energies with explicit time integrator as:

$$\begin{aligned}
 \Delta W_{IC,n+1} + \Delta W_{comp,n+1} + \Delta W_{int,n+1} &= \Delta W_{ext,n+1} + \Delta W_{IC,n+1} \\
 \Delta W_{kin,n+1} &= \left[\frac{1}{2} \dot{U}^T \underline{M} \dot{U} \right]_n^{n+1} \\
 \Delta W_{comp,n+1} &= \left[-\frac{1}{8} W^T \underline{M} W \right]_n^{n+1} \\
 \Delta W_{ext,n+1} &= [U]^T \langle F_{ext,n} \rangle \\
 \Delta W_{int,n+1} &= [U]^T \langle F_{int,n} \rangle \\
 \Delta W_{IC,n+1} &= \frac{[U]^T}{\Delta t} \langle L_n \rangle
 \end{aligned} \tag{7}$$

Where the notations $[.]$ and $\langle . \rangle$ denote the increment and the mean value.

NUMERICAL EXAMPLES

Implementation of the numerical example

The IRSN model was developed with MATLAB [18]. To simplify the model, only two subsets are considered (cf. figure 1), the shock absorber and the lid. The shock absorber subset gathers the mass of the packaging (mainly shock absorber and body) and the mechanical behaviour of the damping material (wood in the case studied in this paper). The lid subset gathers the mass of the lid and content and the elastic behaviour of the screws. All parameters, derived from existing package, are summarized in table 1.

Table 1: Parameters of the modelled package

Subset	Parameter	Value
Shock absorber	Packaging mass (M_s)	15 200 kg
	Shock absorber diameter (D_s)	2 208 mm
	Shock absorber height (H_s)	368 mm
	Shock absorber Young's modulus (E_s)	500 MPa
	Yield strength (Re_s)	10 MPa
	Shock absorber damping ratio (ζ_s)	0.01
Lid	Content mass (M_c)	3 175 kg
	Lid mass (M_l)	1 260 kg
	Screw type (D_l)	M36
	Screw step (P_l)	4
	Screw Young's modulus (E_l)	210 GPa
	Screw quantity (N_l)	39
	Screw clearance hole height (H_l)	60
Initial parameters	Screw damping ratio (ζ_l)	0.01
	Gap (δ)	40 mm
	Initial drop height (H_{init})	9 m

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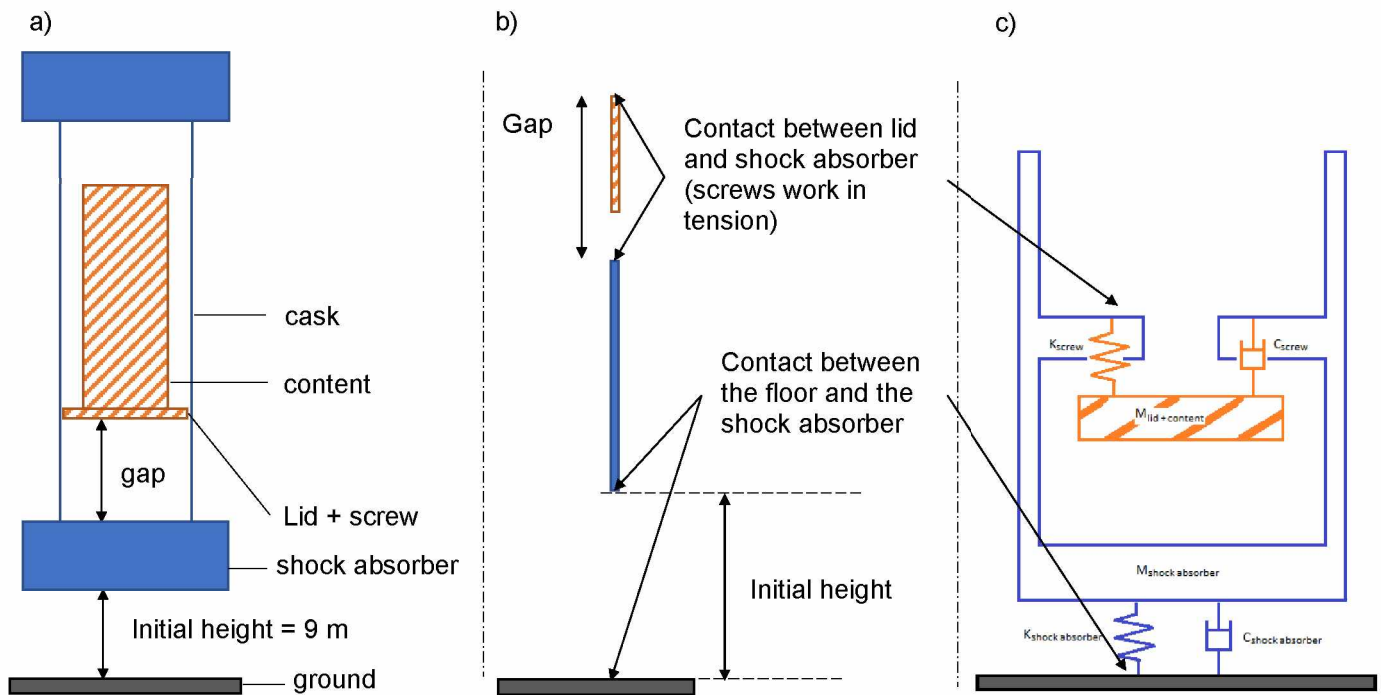


Figure 1: a) simplified drawing of a transport package consisting of two subsets, b) IRSN simplified model developed in MATLAB consisting in two subsets and c) schematic springs/dampers representation of the model during contact (striped orange: lid, screw and content – blue: packaging and shock absorber)

The lid was modeled by a 3-node beam and the shock absorber was modeled with a 3-node beam. Their respective stiffnesses K_l (only due the screws) and K_s are expressed as:

$$\underline{K_s} = E_s \times \frac{\pi \frac{D_s^2}{4}}{H_s} \quad (8)$$

$$\underline{K_l} = N_l \times E_l \times \frac{\pi \frac{(D_l - (0.9381 \times P_l))^2}{4}}{H_l} \quad (9)$$

Finally, the initial speed is null with an initial height of the package of 9 m.

Results

The position of each node for a step time of $0.5 dt_{critical}$, the velocity and acceleration of the node at the center of the shock absorber and the screws during the 9 m drop are shown in figure 2. On the "position" sub-figure, from 0 to 1.35 s, the package is in free fall (constant acceleration of $9.81 \text{ m}\cdot\text{s}^{-2}$ and linear increase of the velocity until it reaches about $13.5 \text{ m}\cdot\text{s}^{-1}$). The shock absorber then comes into contact with the ground (blue dotted curve and black line) for a duration of about 40 ms and then bounces back to about 1.5 m. During the contact of the shock absorber with the ground, the content continues falling until it impacts the lid at about 1.36 s (blue dotted curve and red dotted curve). This is highlighted on the velocity curve and the acceleration curve (red curve).

The results indicate a maximum acceleration of 3100 g for the shock absorber and 610 g for the screws. The plastic strain of the shock absorber is 21.8 % and the maximum elastic strain of the screws is 2.3 %.

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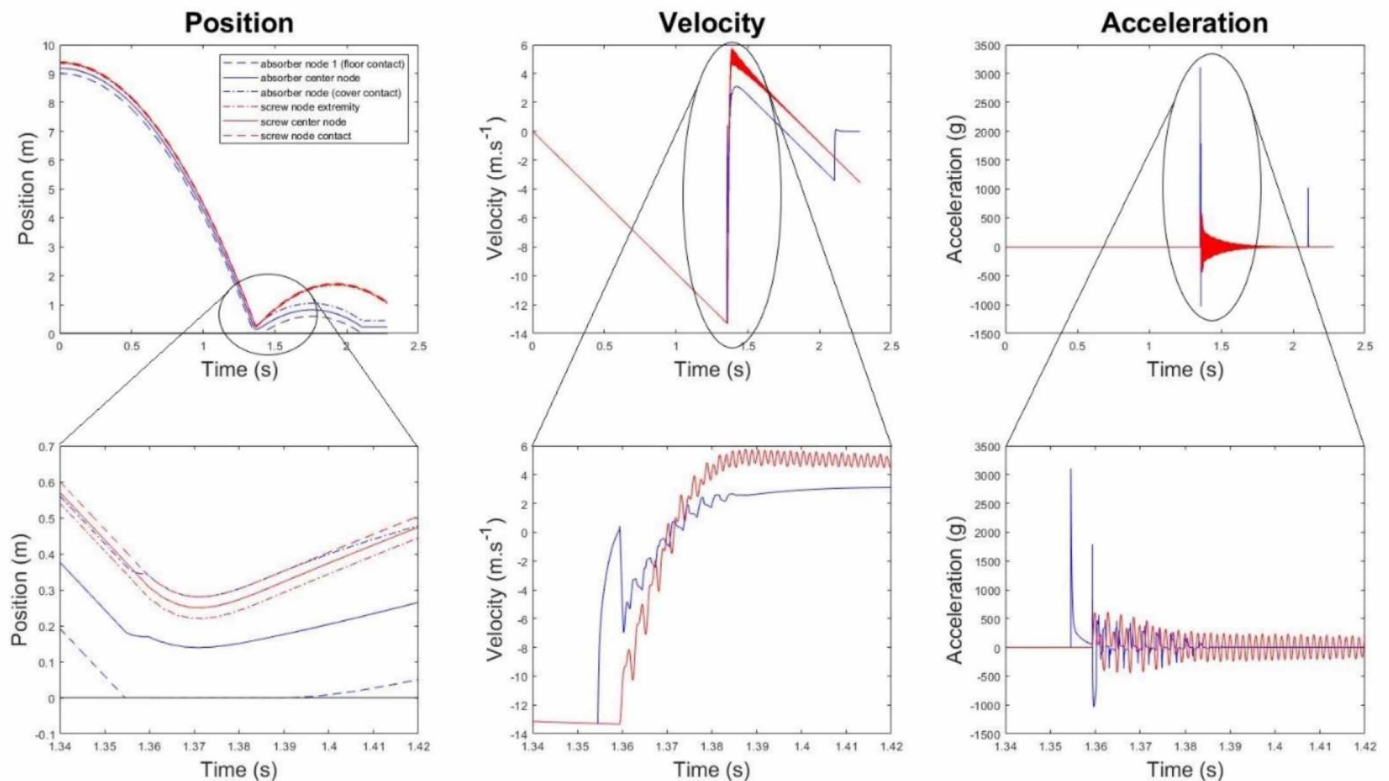


Figure 2: Position, velocity and acceleration during the drop test with simulated gap between the content and the lid of 40 mm

Figure 3 shows the evolution of the different parameters useful for the analysis of the delayed impact as a function of the gap between 0 and 800 mm with a time step of $0.01 dT_{critical}$.

Figure 3a and figure 3b show the evolution of the maximum plasticity and the maximum acceleration of the impact absorber. Figure 3c and Figure 3d show the maximum elastic strain and the maximum acceleration of the screws. Figure 3b shows that for all gaps, the maximum acceleration of the shock absorber is 3100 g and the corresponding plasticity decreases logarithmically according to the gap. This lack of variation can be explained by the fact that, on the one hand the contents represent only 15 % of the total mass of the package, on the other hand the point of interest of this curve, *i.e.* the center of the shock absorber, is rather far from the points of contact, which tends to smoothen the acceleration curve.

Figure 3c and figure 3d show a very similar pattern. The acceleration and the strain increase until reaching a maximum (respectively 650 g and 2.7 %). This local maximum is reached when the delayed impact occurs at the maximum plastic strain of the shock absorber (figure 4a), *i.e.* at the moment from which its springback begins (upward movement). A second peak occurs around a gap of 400 mm; this is reached when the delayed impact occurs when the rebound of the shock absorber begins, which fits the moment when the energy is maximum (figure 4b).

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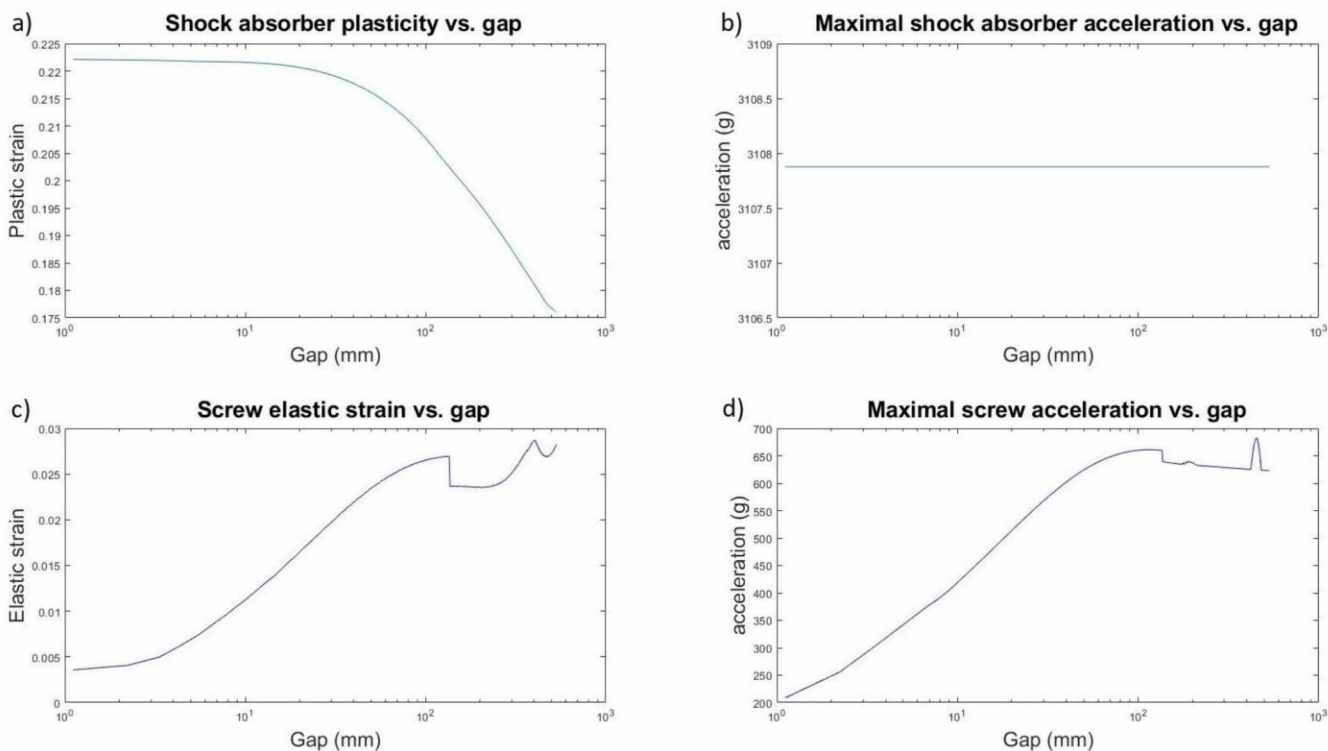


Figure 3: Influence of the gap for a) the shock absorber plasticity, b) the shock absorber acceleration, c) the screw strain and d) the screw acceleration

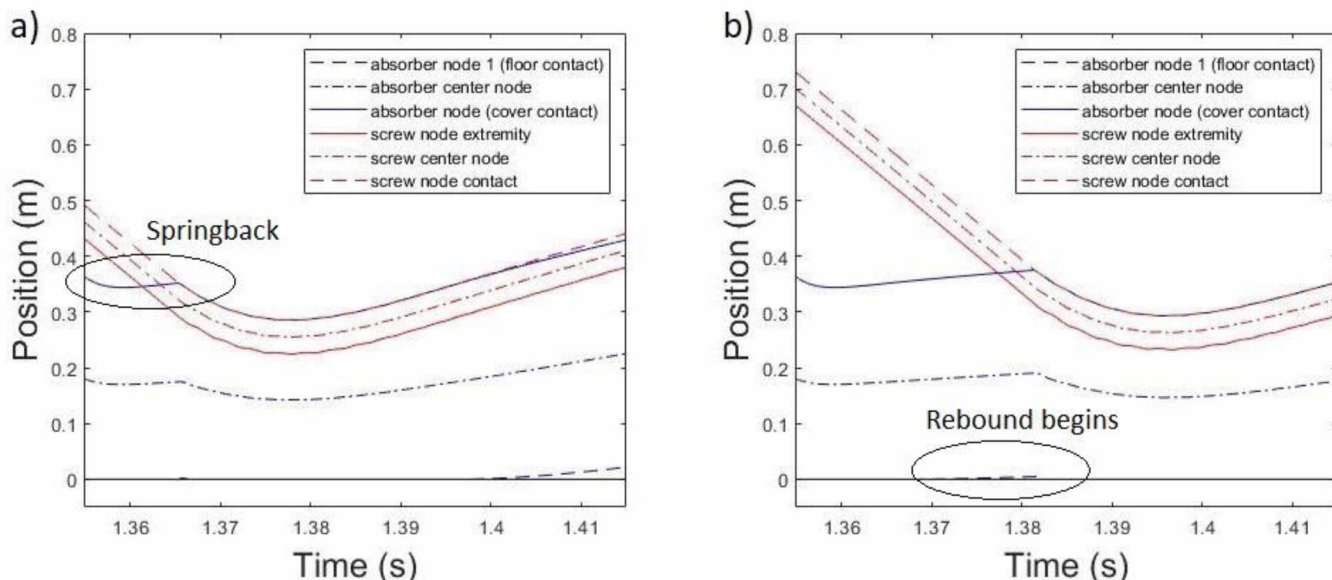


Figure 4: a) Position over time for each node for a gap of 131 mm (first peak) and b) for a gap of 404 mm (second peak)

Sensitivity analysis

In order to illustrate the global sensitivity and the correlation for each parameter presented in table 1 on the screw acceleration parameters in figure 5, IRSN derived a sensitivity matrix (\underline{S}) such as:

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$$S_{ij} = \frac{(\{|\theta_i|\}^t \cdot \{|\theta_j|\})^{1/2}}{N_t} \quad (10)$$

where N_t is the number of step time and $\{\theta_i\}$ is, for a 1% variation of a given parameter, either a scalar of the variation of the maximal acceleration of screws, or the vector gathering one nodal acceleration variation over time. A high level in the sensitivity matrix represents a significant influence of the observed parameter. Furthermore, a high level of off-diagonal terms suggests inter-dependencies between parameters as well. Figure 5a presents the sensitivity on the maximal acceleration and in figure 5b show the sensitivity on the acceleration over time for a simulation from 1.35 s to 1.5 s with a fix step time of $1,7e^{-6}$ s (lower than the critical step time).

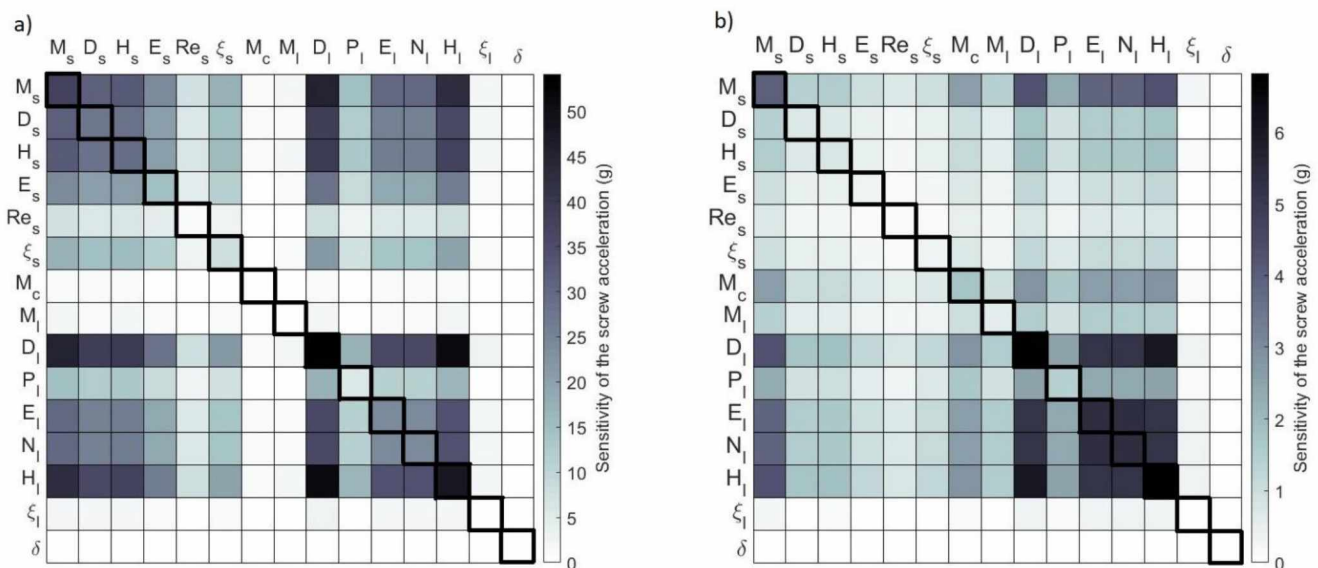


Figure 5: a) Sensitivity matrix for each parameter from table 1 on the screw maximal acceleration and b) on the screw acceleration over the time

Focusing on the diagonal of figure 5a, the results indicate that a 1% variation in gap, screw damping, content mass and lid mass have little influence on screw maximal acceleration. On the contrary, a variation of 1% of the height of the screw clearance hole, the diameter of the screws, the number of screws, the stiffness of the screws, the mass, the diameter and the height of the shock absorber have a significant influence. The off-diagonal terms, illustrating the correlation between the different terms on screw maximal acceleration, have few interests.

Figure 5b is an interesting tool to be used when proceeding to a rescaling on an experimental signal for example: it outlines the consequences of the variations on the whole acceleration signal. In the present case, the sensitive parameters are related to the variations of screw characteristics and to the mass of the shock absorber, and the least significant ones are related to the damping and gap variations. Finally, it shows that the variation of the mass of the cover and the contents have a significant effect on the whole acceleration signal, but not on the maximum acceleration as shown in figure 5a. In the same way, it can also be noted that the variation in the characteristics of the shock absorber is of second order on the whole signal, but more significant on the maximal acceleration.

Focusing on the off-diagonal terms, it can be observed that the screw parameters are strongly correlated. This high sensitivity means that their variation will have the same effect on the acceleration signal. The mass of the shock absorber also presents a non-negligible correlation with the parameters of the screws on the acceleration signal.

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Therefore, this analysis highlights that, in the context of safety assessment or in the context of an analysis on the assembly, some parameters will require more accurate definition because their variation may have significant consequences on the resulting strains in the screws. However, since parameters can also have a non-linear effect on the results, it will be necessary to extend this sensitivity analysis to other numerical examples. IRSN will also extend this analysis to the evaluation of other characteristics on the whole fall of the package in order to refine the study of the correlation between each parameter.

CONCLUSION

This paper presents preliminary IRSN results on the development of a tool to quickly obtain orders of magnitude on strains and accelerations that can occur during a delayed impact of the content on the lid of a package. In particular, the possible influence of a small variation of the input parameters taken into account on these characteristics is illustrated through a sensitivity analysis.

Thus far, the IRSN numerical model considers only two beams representing two subsets consisting of, on the one hand the shock absorber with the mass of the packaging, on the other hand the screws with the mass of the lid and content. Two contacts are considered: one between the ground and the shock absorber and between the two beams. This modeling implies that:

- the boundary effects of the lid are not considered (diameter of the shell smaller than that of the lid, in particular),
- the bending of the lid and the stiffness of the contents is not considered,
- the contact between the lid and the contents is not considered,
- the lid is considered homogeneous with a single material,
- the effect of the discretization size is not studied.

The simplified assumptions mentioned above will be investigated to appreciate the limitation of the model. It is important to keep in mind that the model must allow for a relatively fine order of magnitude within the framework of the safety assessments carried out at IRSN in order to clarify the consequences of the hypotheses considered by the applicants on the mechanical behavior of the containment system.

More importantly, as several parameters were chosen more or less arbitrarily (e.g. damping parameters and, more generally, mechanical parameters of the shock absorber), IRSN will calibrate then validate its model according to experimental or numerical data.

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